Improving Quantum Sensing with Quantum Computers

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QUANTUM METROLOGY

metrology
/mɪˈtrɒlədʒi/
noun
the scientific study of measurement.

- Goal: Best estimate of physical parameters
- Sensitivity enhancement with quantum effects

Probe State → Encoding Evolution → Measurement --> Output Distribution

$$\rho \longrightarrow \mathcal{E}(\phi) \longrightarrow \{\Pi_l\} \dashrightarrow p_l(\phi)$$

OUR COST FUNCTION

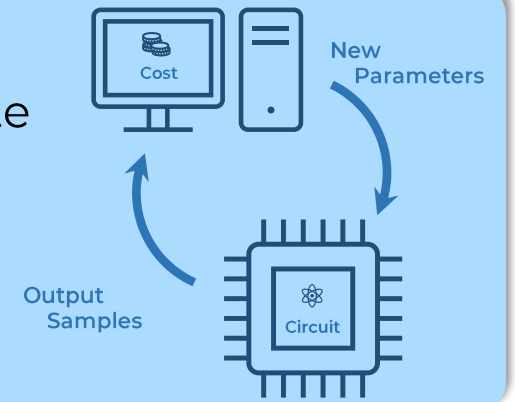
- Maximal precision given by Cramér-Rao bound

Estimator Error $\mathrm{MSE}(\hat{\varphi}) \geq \mathrm{Tr}\{I_f^{-1}\}$ Information Content

- Needs Classical Fisher Information Matrix
- Calculate via parameter-shift rule

VARIATIONAL ALGORITHMS

- Parametrize Quantum State on Device
- Sample Output
- Update parameters to minimize Cost



KEY FACTS AND ADVANTAGES

- Use Classical Fisher Information, not Quantum
- Optimize full sensing protocol, not only probe
- Allow for multi-parameter sensing
- Allow for arbitrary post-processing

PAPER DEMO SLIDES

arxiv:2006.06303

A variational toolbox for quantum multi-parameter estimation

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(Dated: June 11, 2020)

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Quantum Metrology

Physical quantities (magnetic fields, energies, …) need to be **measured** accurately

Study how **quantum effects** can help

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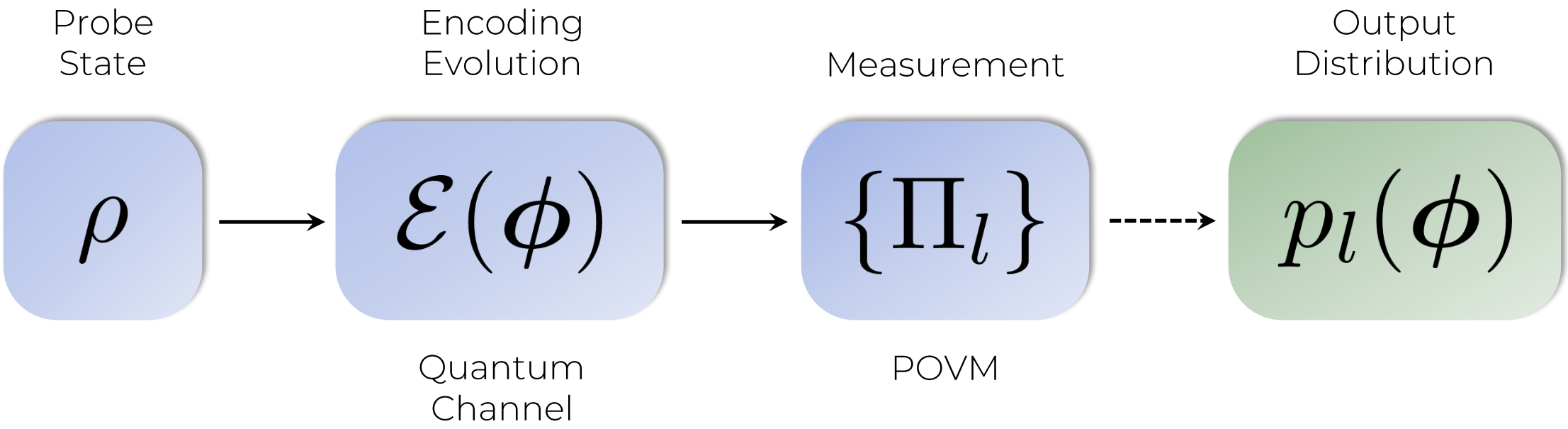


Cramér-Rao Bound

Task: Compute an **estimator** from samples of the output distribution

$$p_l(\boldsymbol{\phi}) \;\rightarrow\; \hat{\boldsymbol{\varphi}} : \mathbb{E}\{\hat{\boldsymbol{\varphi}}\} = \boldsymbol{\phi}$$

Precision from n samples is limited by **Cramér-Rao bound**

$$\mathrm{Cov}(\hat{\boldsymbol{\varphi}}) \geq \frac{1}{n} I_{\boldsymbol{\phi}}^{-1}(\mathrm{POVM}) \geq \frac{1}{n} \mathcal{F}_{\boldsymbol{\phi}}^{-1}$$

$$\mathrm{Tr}\{\mathrm{Cov}(\hat{\boldsymbol{\varphi}})\} = \mathrm{MSE}(\hat{\boldsymbol{\varphi}})$$

Recap #1

Sensing amounts to estimating the underlying physical parameters from a classical probability distribution

Estimation precision is limited by the Cramér-Rao bound

Attainble precision is quantified by the classical Fisher information

The quantum Fisher information bounds the attainable Fisher information

→ Classical Fisher information should be used to judge sensing quality!

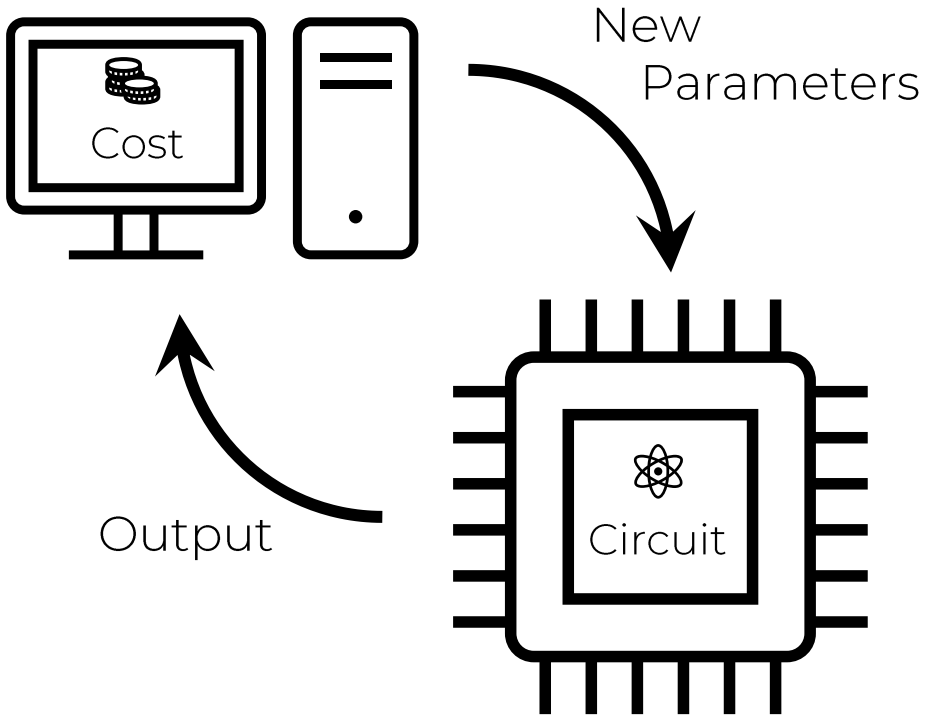
Optimal Metrology

We need to find optimal probes and measurements

Complicated under noise and device limitations

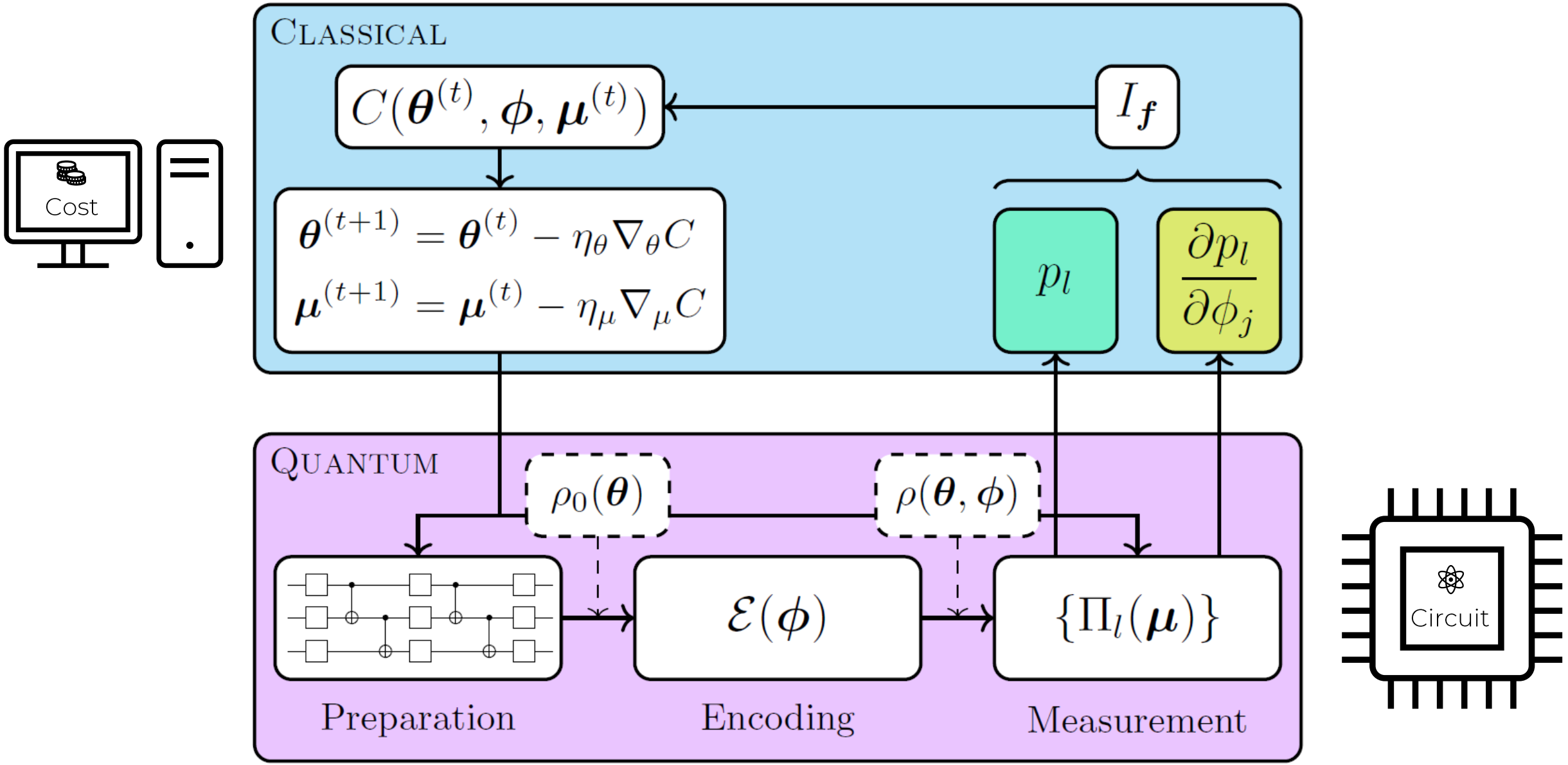
NISQ techniques come to the rescue:
use variational approaches

Prior work[1,2] focused on probes for single-parameter metrology and surrogates for the Quantum Fisher Information



[1]Kaubruegger, Raphael, et al. *Physical Review Letters* 123.26 (2019): 260505.
[2]Koczor, Bálint, et al. "Variational-State Quantum Metrology." *New Journal of Physics* (2020).

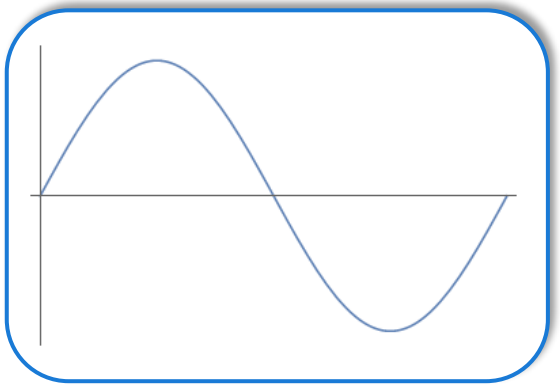
The Algorithm



Calculation of Fisher Information

Fisher Information Matrix w.r.t. the physical parameters

$$[I_{\boldsymbol{\phi}}]_{jk} = \sum_l \frac{1}{p_l} \frac{\partial p_l}{\partial \phi_j} \frac{\partial p_l}{\partial \phi_k}$$



Exploit parameter-shift rule[1,2] to calculate derivatives

$$\partial_j p_l(\boldsymbol{\phi}) = \frac{1}{2}\left[p_l\left(\boldsymbol{\phi} + \frac{\pi}{2}\boldsymbol{e}_j\right) - p_l\left(\boldsymbol{\phi} - \frac{\pi}{2}\boldsymbol{e}_j\right)\right]$$

[1]Schuld, Maria, et al. *Physical Review A* 99.3 (2019): 032331.
[2]Banchi, Leonardo, and Gavin E. Crooks. arXiv preprint arXiv:2005.10299 (2020).

Cost Function

We are usually interested in a function of the parameters:
Exploit transformation rule of Fisher Information Matrix

$$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{\phi}) \;\rightarrow\; I_{\boldsymbol{f}} = J^T I_{\boldsymbol{\phi}} J \qquad J_{jk} = \frac{\partial f_j}{\partial \phi_k}$$

Need a scalar cost function:
Apply weighted trace to both sides of the CRB!

$$\mathrm{Tr}\{W\,\mathrm{Cov}(\hat{\boldsymbol{\varphi}})\} \geq \frac{1}{n}\,\mathrm{Tr}\{W I_{\boldsymbol{f}}^{-1}\} = \frac{1}{n} C_W$$

$$\begin{aligned}\mathrm{Tr}\{W\,\mathrm{Cov}(\hat{\boldsymbol{\varphi}})\} &= \mathrm{MSE}_W(\hat{\boldsymbol{\varphi}}) \\ &= \mathbb{E}\{\langle \hat{\boldsymbol{\varphi}} - \boldsymbol{\phi}, W(\hat{\boldsymbol{\varphi}} - \boldsymbol{\phi})\rangle\}\end{aligned}$$

Recap #2

The classical Fisher information matrix is calculated from the output probabilities and their derivatives w.r.t. the physical parameters

The derivatives can be calculated on the device via the parameter-shift rule

The classical Fisher information matrix w.r.t. post-processed parameters can be computed using the postprocessing‘s Jacobian

The cost function is obtained from a weighted trace of the Cramér-Rao bound

Implementation Prerequisites

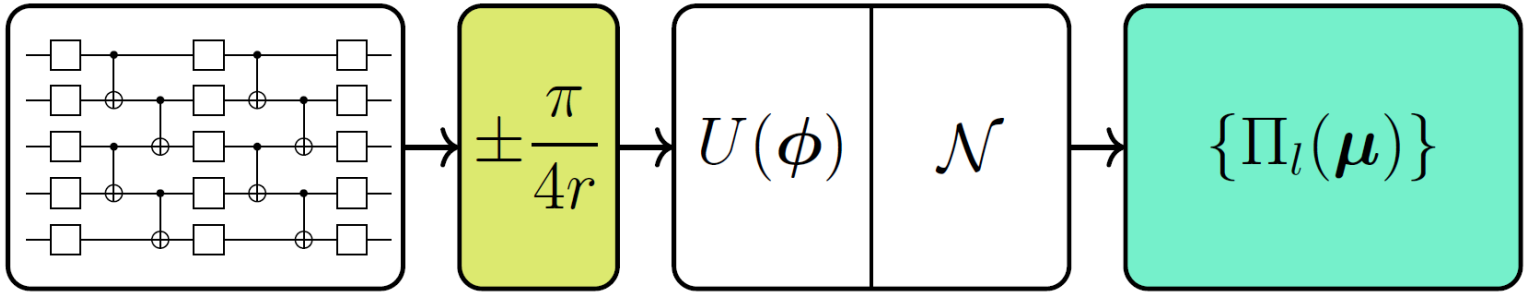
Knowledge about the encoding process and noise sources

Example: Unitary encoding with commuting noise process

$$\mathcal{E}(\boldsymbol{\phi})[\rho_0] = \mathcal{N}[U(\boldsymbol{\phi})\rho_0 U^\dagger(\boldsymbol{\phi})]$$

Ability to manipulate or spoof the physical parameters

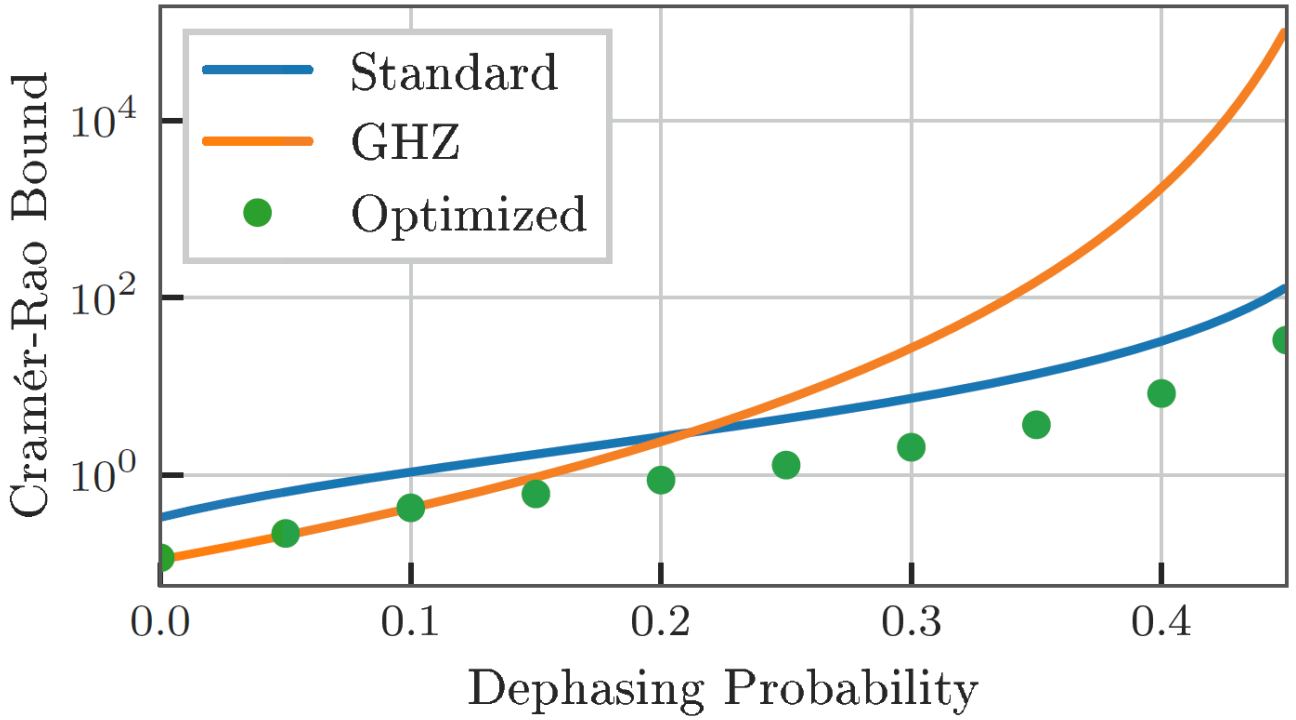
Example: Unitary encoding with phase injection



Numerics: Ramsay Spectroscopy

Task: Estimate the average of three phases under dephasing noise

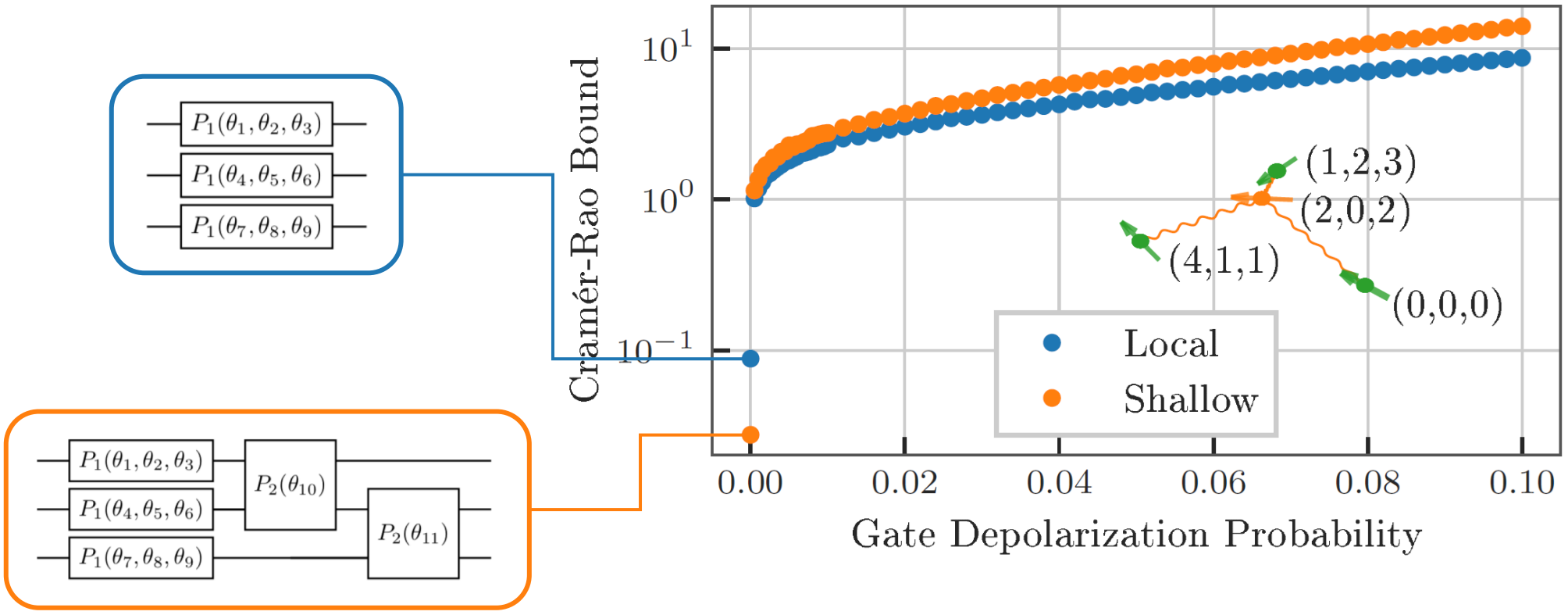
Fixed phase parameters and varied noise level



Numerics: NV Trilateration

Task: Estimate the position of a spin interacting with three sensing spins

Compare local and shallow state preparations for different gate error levels

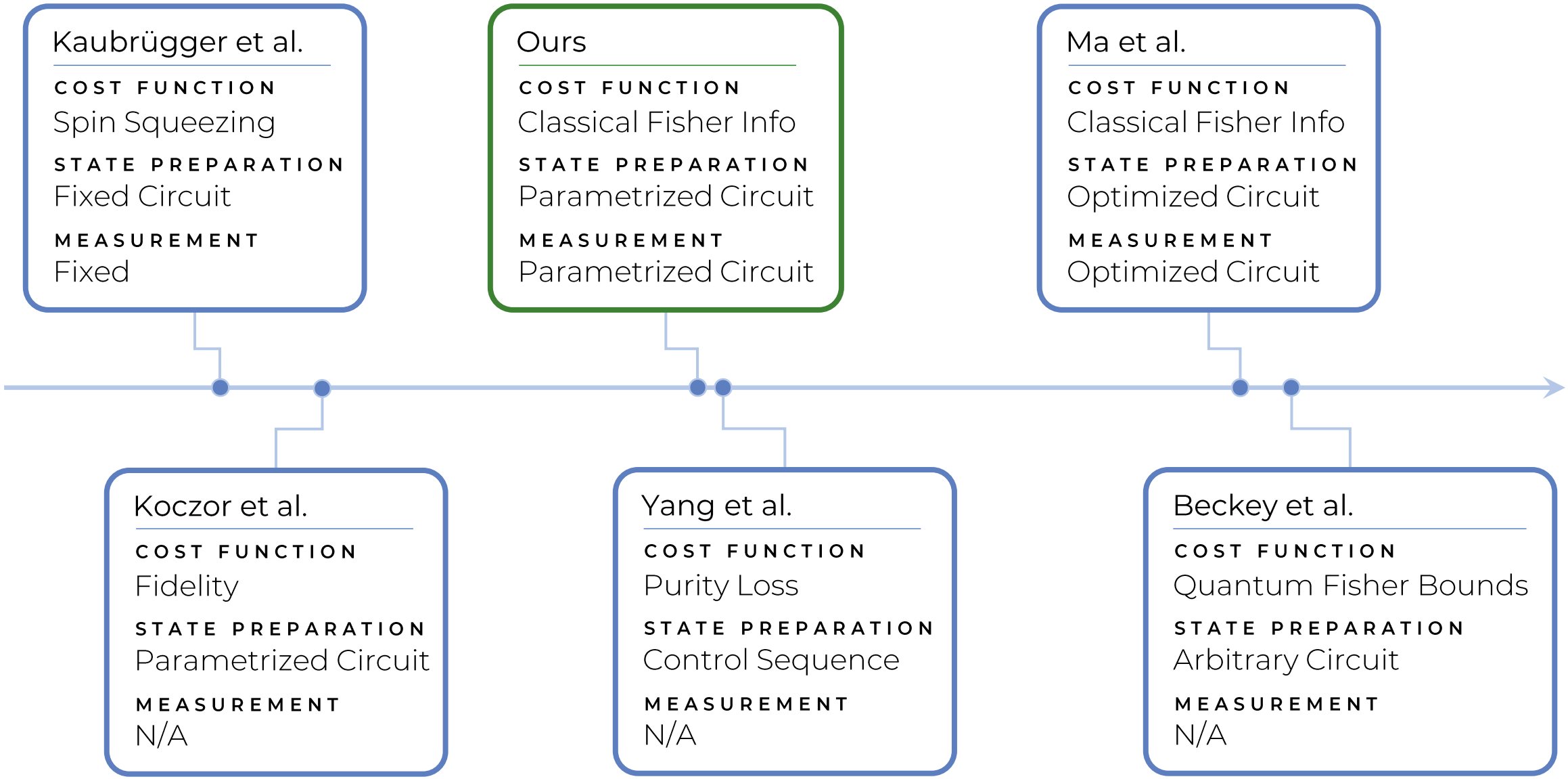


Further Contents

We provide multiple extensions of the algorithm, for example including prior knowlege (Bayesian approach)

We provide a parameter-shift rule for noise channels

We give details on the implementation of parameter-shift rules

The Algorithm Landscape

Single Parameter Multiparameter