WRACHTRUP GROUP SEMINAR

Improving Quantum Sensing with Variational Methods

JOHANNES JAKOB MEYER, FU BERLIN

arxiv:2006.06303

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A variational toolbox for quantum multi-parameter estimation

Johannes Jakob Meyer,[1] Johannes Borregaard,[2,3] and Jens Eisert[1]

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[2] *Qutech and Kavli Institute of Nanoscience, Delft University of Technology, 2628 CJ Delft, The Netherlands*
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(Dated: June 11, 2020)

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Quantum Metrology

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Physical quantities (magnetic fields, energies, ...)
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noun

the scientific study of measurement.

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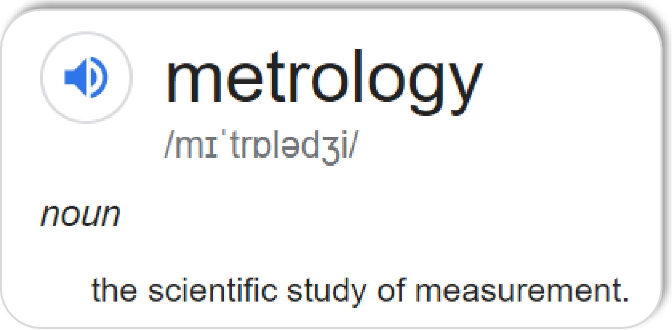
Study how **quantum effects** can help



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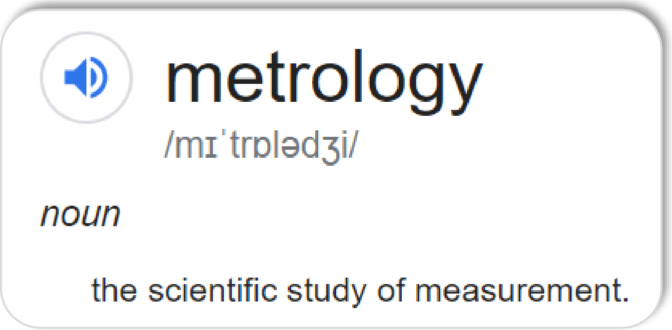

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Probe
State

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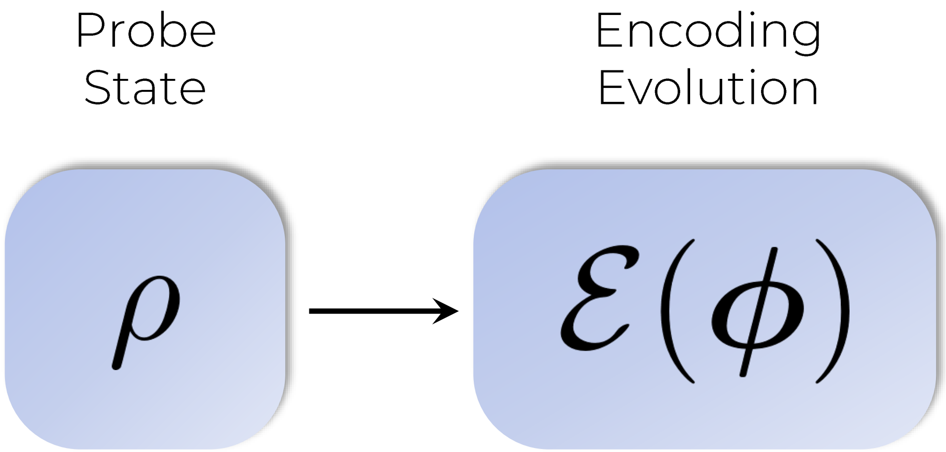

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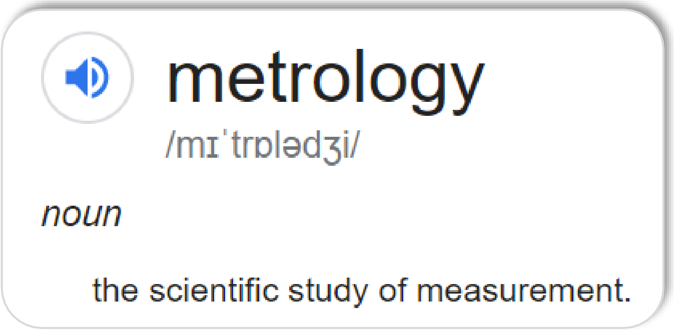

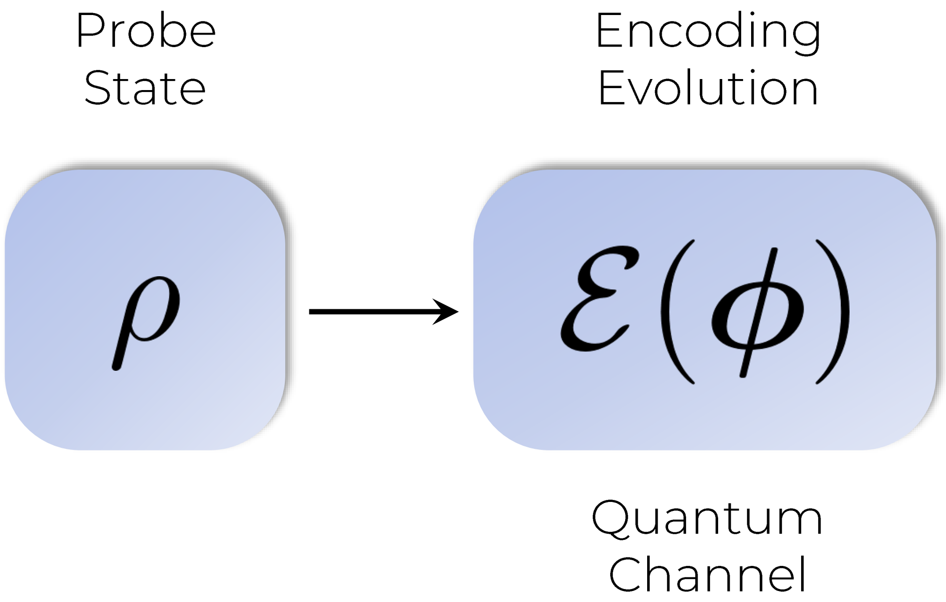

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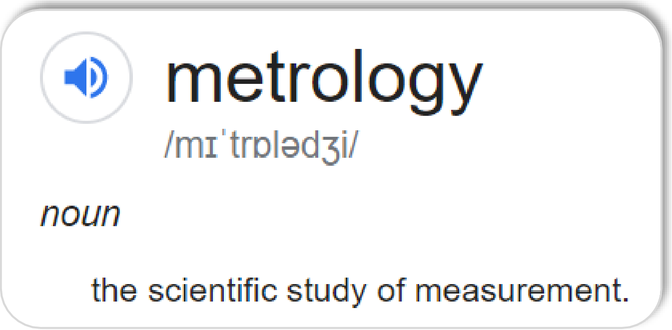

Probe State	Encoding Evolution	Measurement

$$\rho \longrightarrow \mathcal{E}(\phi) \longrightarrow \{\Pi_l\}$$

Quantum Channel

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Probe State	Encoding Evolution	Measurement	Output Distribution
ρ →	$\mathcal{E}(\phi)$ →	$\{\Pi_l\}$ ⇢	$p_l(\phi)$
	Quantum Channel	POVM	

Cramér-Rao Bound

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Task: Compute an **estimator** from samples of the output distribution

$$p_l(\boldsymbol{\phi}) \;\to\; \hat{\boldsymbol{\varphi}} \colon \mathbb{E}\{\hat{\boldsymbol{\varphi}}\} = \boldsymbol{\phi}$$

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Pr $\boxed{\mathrm{Tr}\{\mathrm{Cov}(\hat{\varphi})\} = \mathrm{MSE}(\hat{\varphi})}$ **Cramér-Rao bound**

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Recap #1

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The quantum Fisher information bounds the attainable Fisher information

→ Classical Fisher information should be used to judge sensing quality!

Optimal Metrology

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We need to find optimal probes and measurements

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Complicated under noise and device limitations

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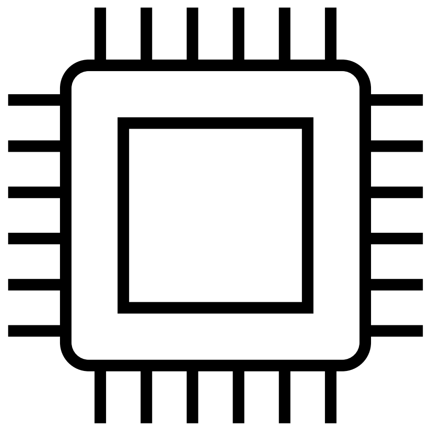

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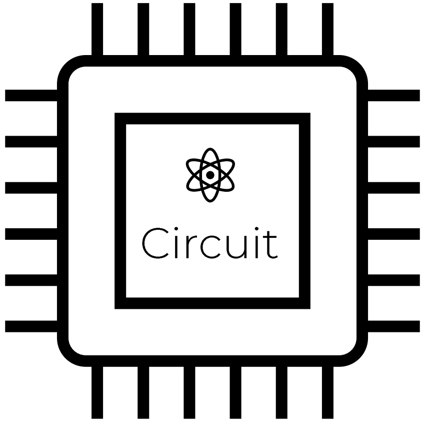

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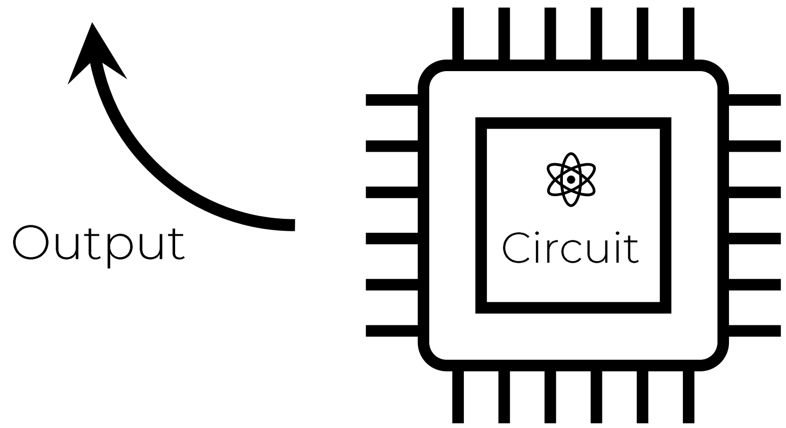

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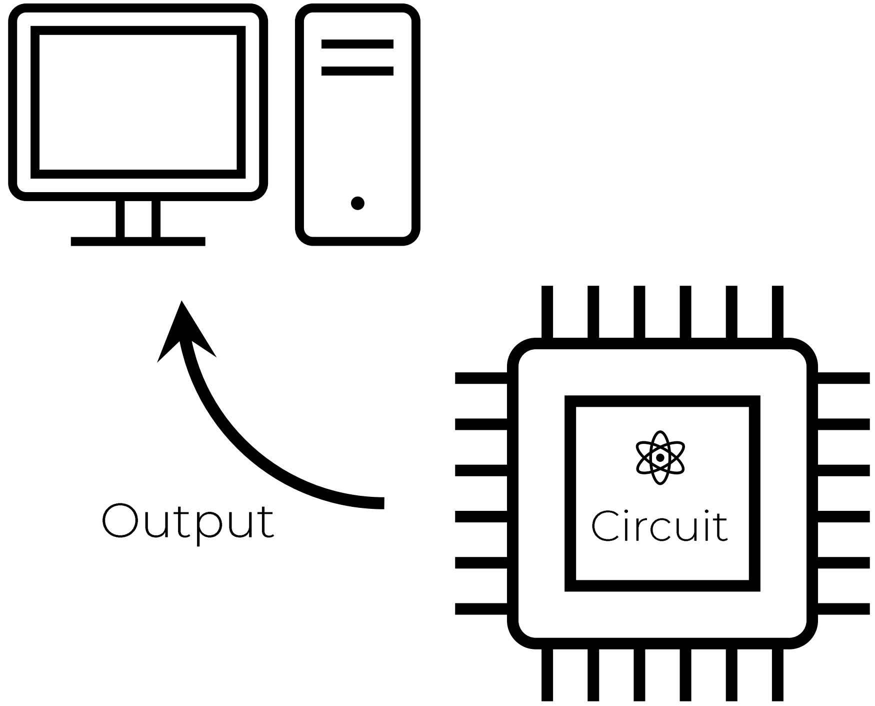

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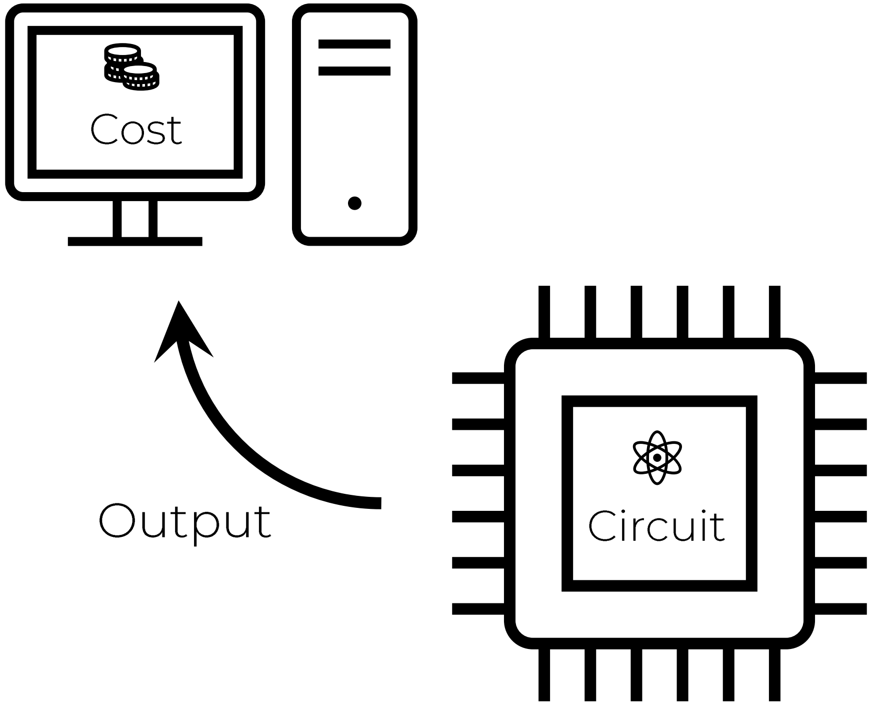

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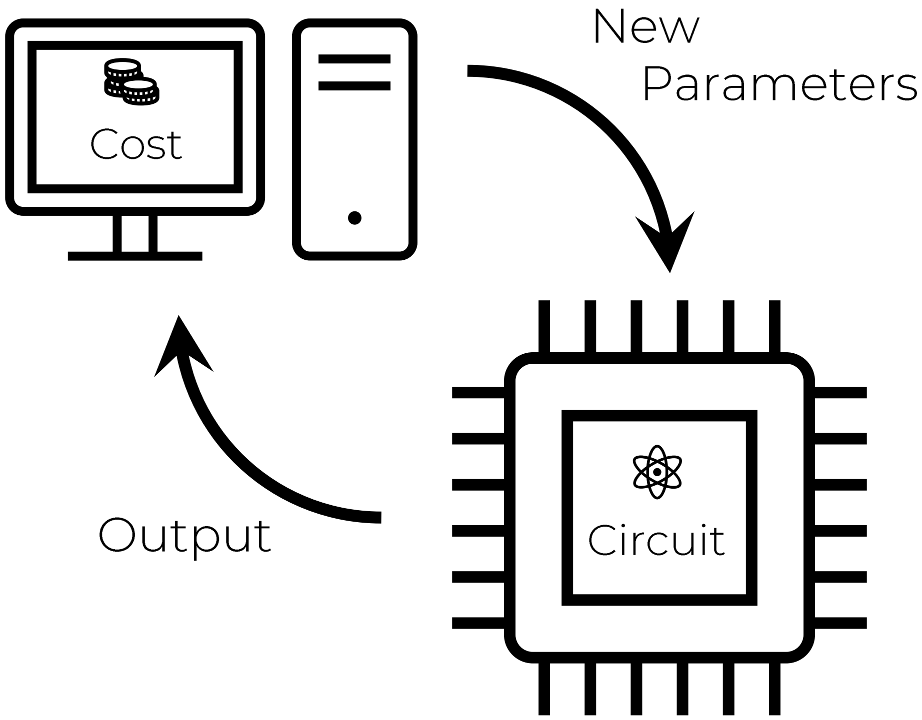

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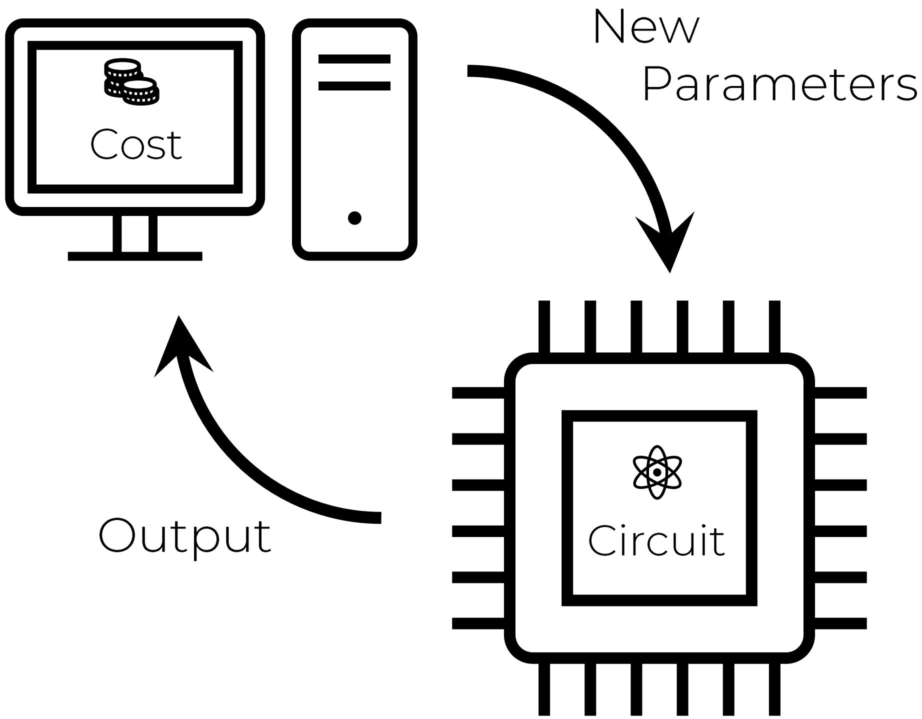

Prior work[1,2] focused on probes for single-parameter metrology and surrogates for the Quantum Fisher Information

[1]Kaubruegger, Raphael, et al. *Physical Review Letters* 123.26 (2019): 260505.
[2]Koczor, Bálint, et al. "Variational-State Quantum Metrology." *New Journal of Physics* (2020).

The Algorithm

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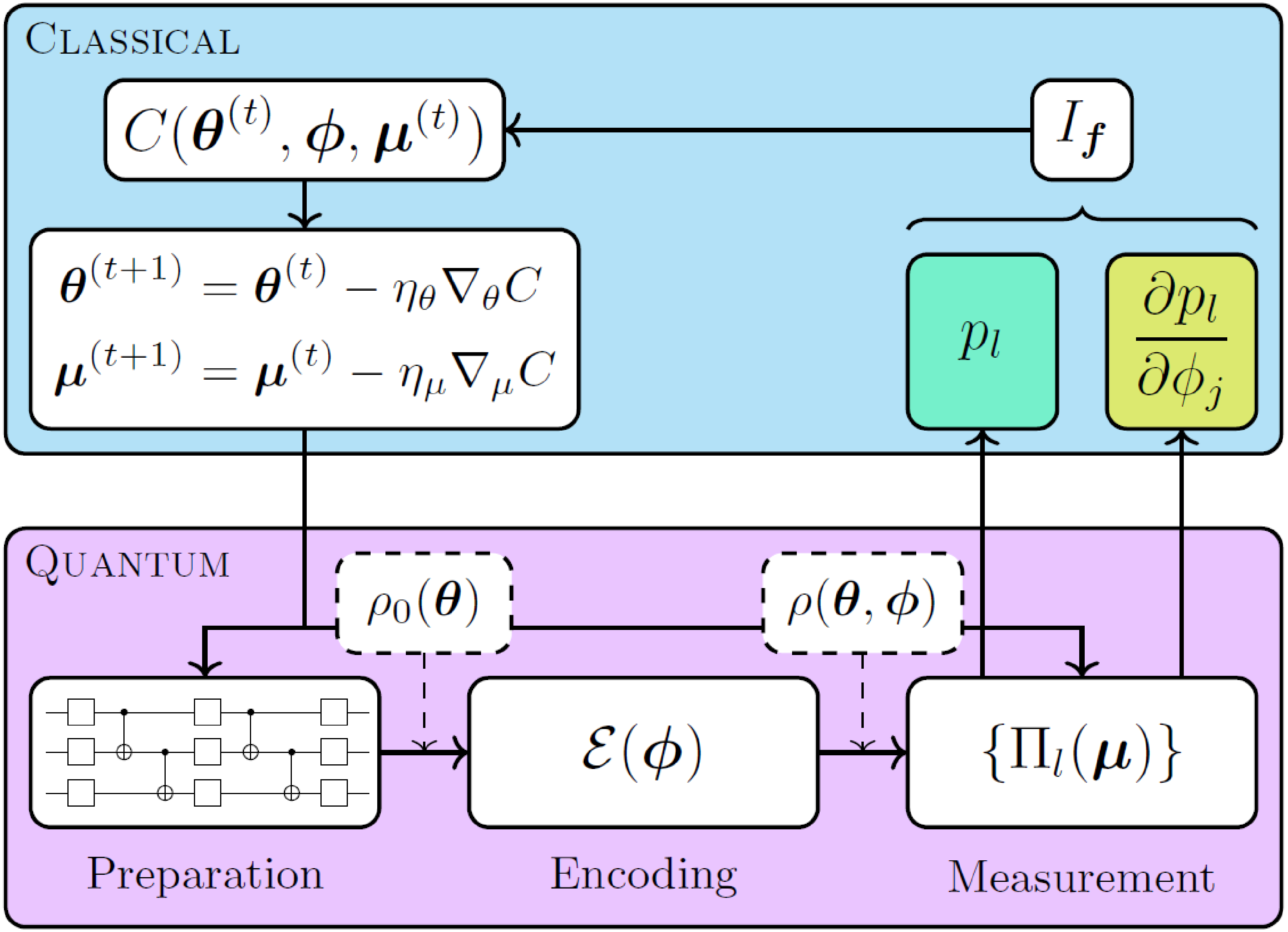

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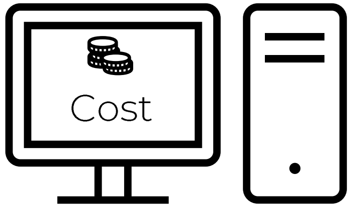

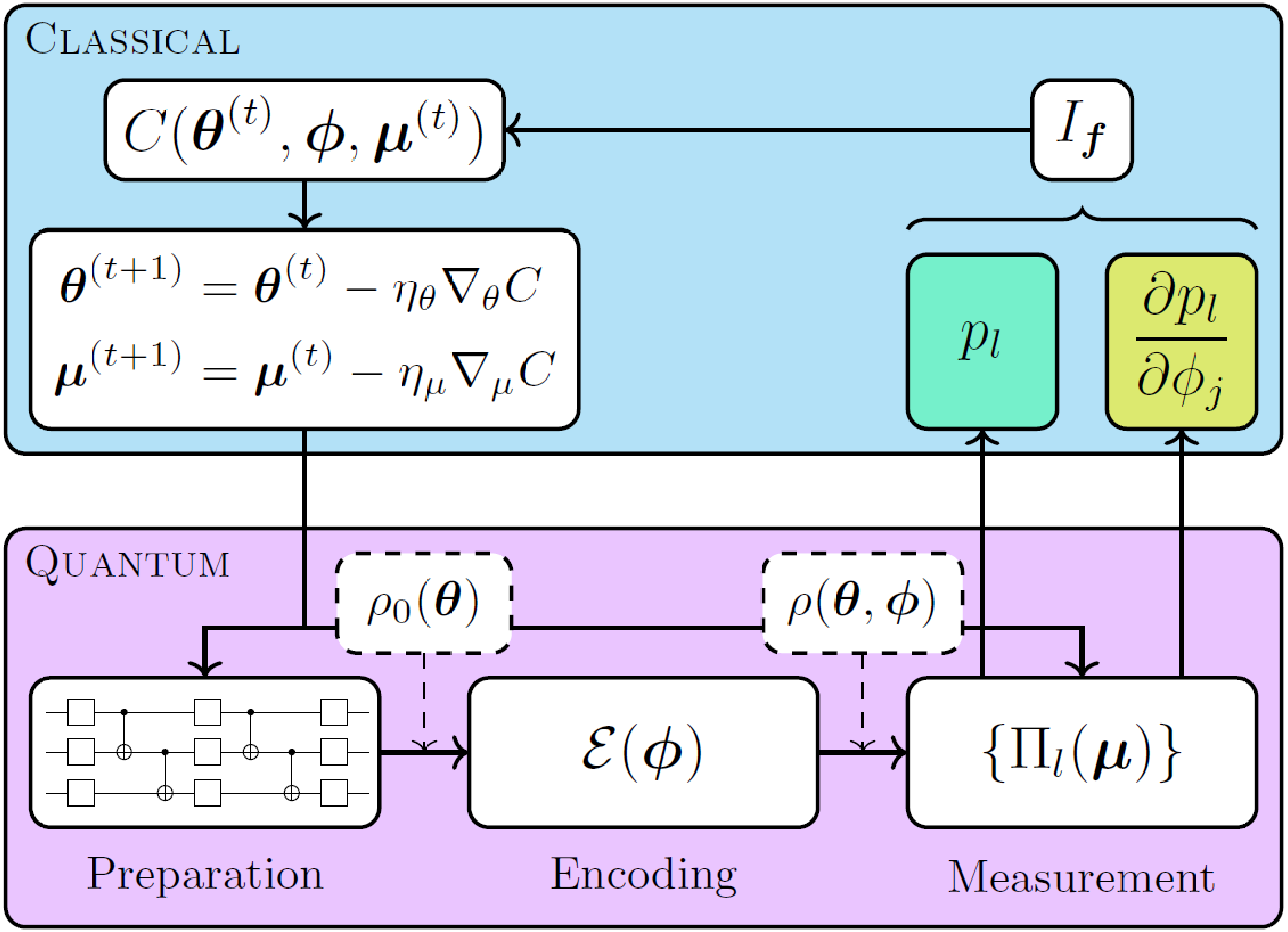

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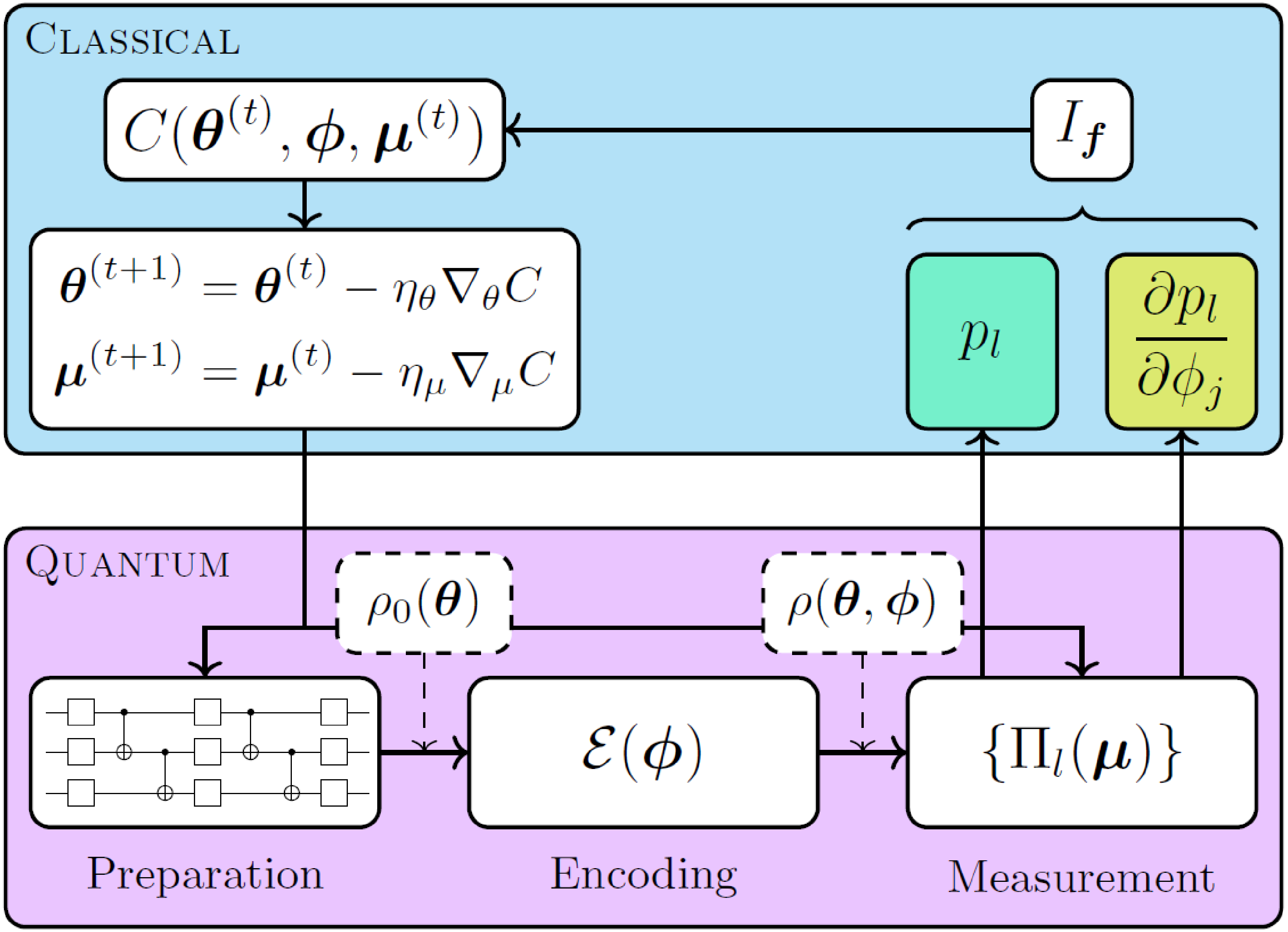

Calculation of Fisher Information

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Fisher Information Matrix w.r.t. the physical parameters

$$[I_{\boldsymbol{\phi}}]_{jk} = \sum_{l} \frac{1}{p_l} \frac{\partial p_l}{\partial \phi_j} \frac{\partial p_l}{\partial \phi_k}$$

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Exploit parameter-shift rule[1,2] to calculate derivatives

$$\partial_j p_l(\boldsymbol{\phi}) = \frac{1}{2} \left[p_l \left(\boldsymbol{\phi} + \frac{\pi}{2} \boldsymbol{e}_j \right) - p_l \left(\boldsymbol{\phi} - \frac{\pi}{2} \boldsymbol{e}_j \right) \right]$$

[1] Schuld, Maria, et al. *Physical Review A* 99.3 (2019): 032331.
[2] Banchi, Leonardo, and Gavin E. Crooks. arXiv preprint arXiv:2005.10299 (2020).

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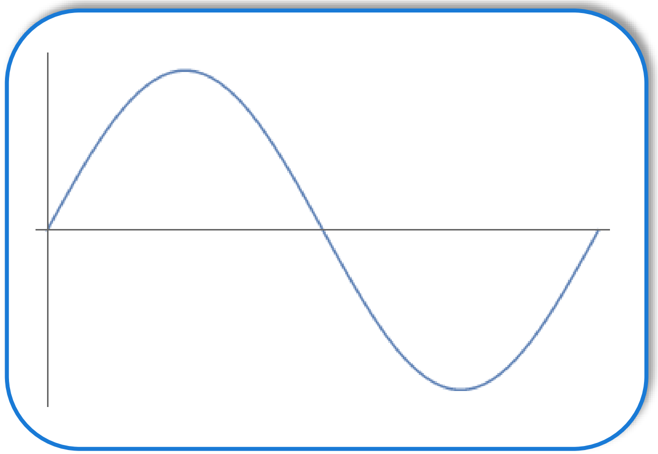

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Apply weighted trace to both sides of the CRB!

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$$\boxed{\begin{aligned}\mathrm{Tr}\{W\,\mathrm{Cov}(\hat{\boldsymbol{\varphi}})\} &= \mathrm{MSE}_W(\hat{\boldsymbol{\varphi}}) \\ &= \mathbb{E}\{\langle \hat{\boldsymbol{\varphi}} - \boldsymbol{\phi}, W(\hat{\boldsymbol{\varphi}} - \boldsymbol{\phi})\rangle\}\end{aligned}}$$

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The cost function is obtained from a weighted trace of the Cramér-Rao bound

Implementation Prerequisites

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Knowledge about the encoding process and noise sources

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Example: Unitary encoding with commuting noise process

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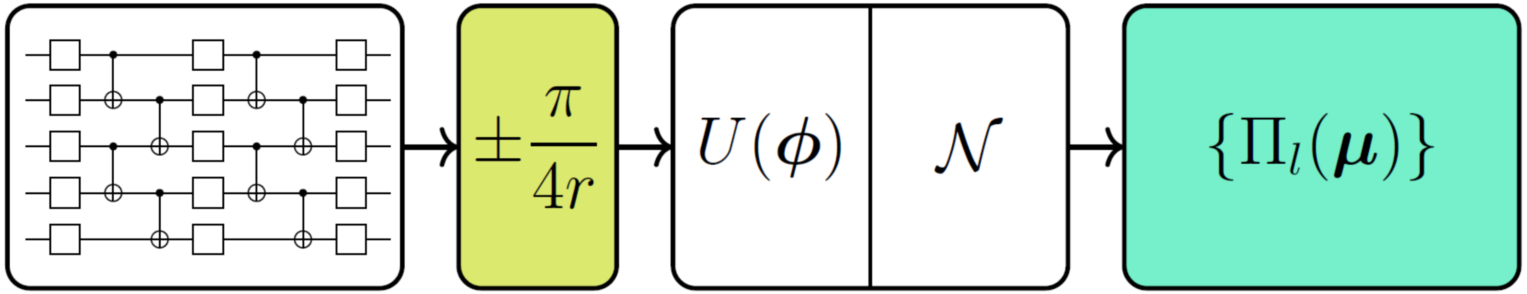
Ability to manipulate or spoof the physical parameters

Example: Unitary encoding with phase injection



Numerics: Ramsay Spectroscopy

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Task: Estimate the average of three phases under dephasing noise

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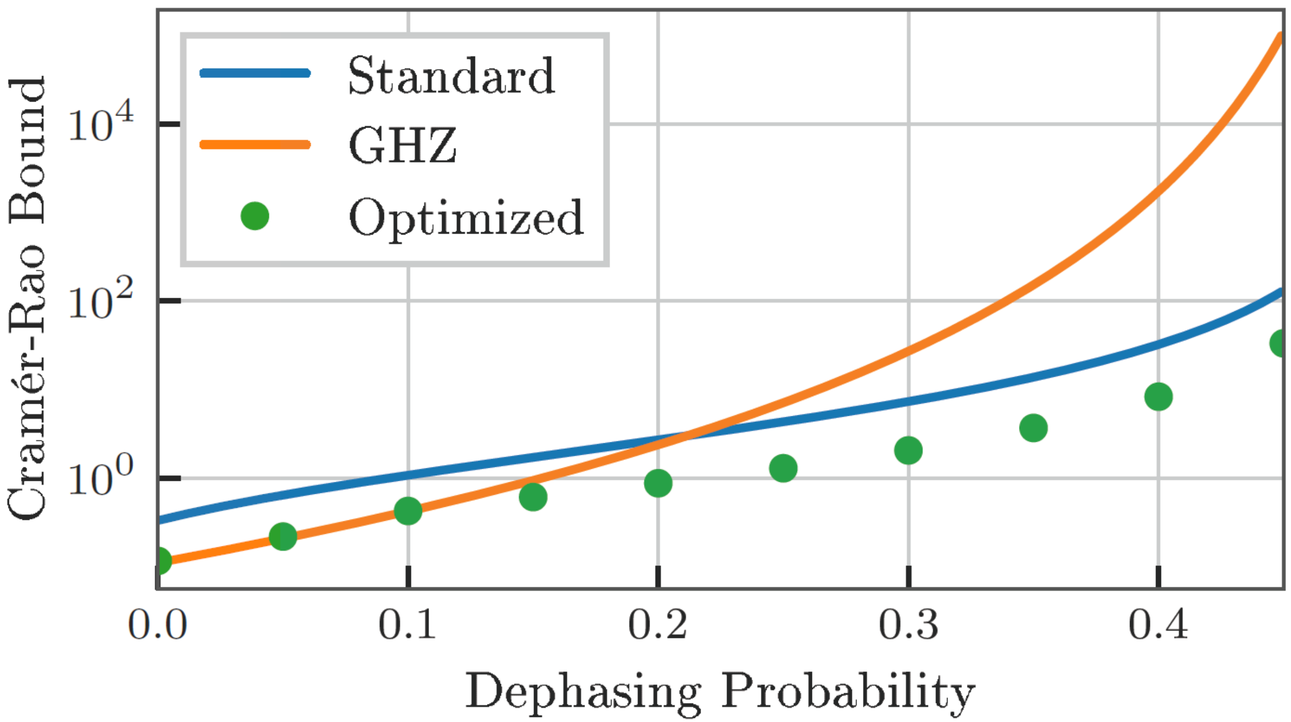

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Compare local and shallow state preparations for different gate error levels

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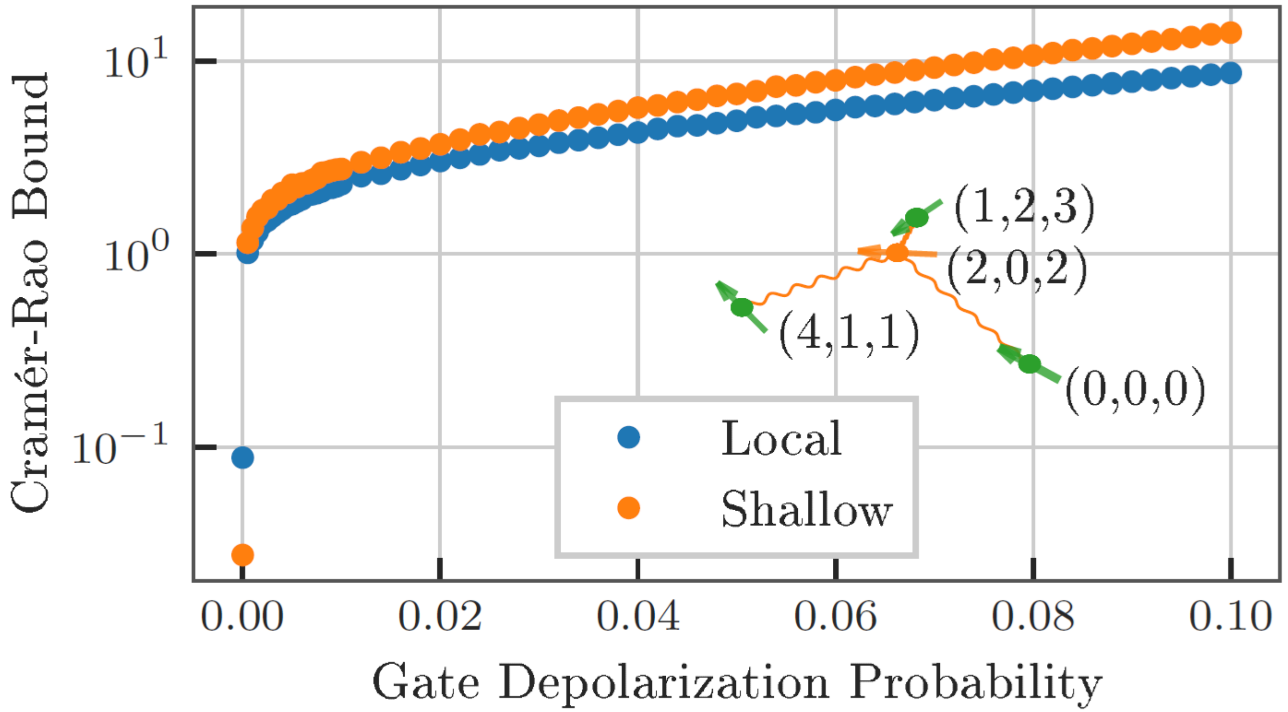

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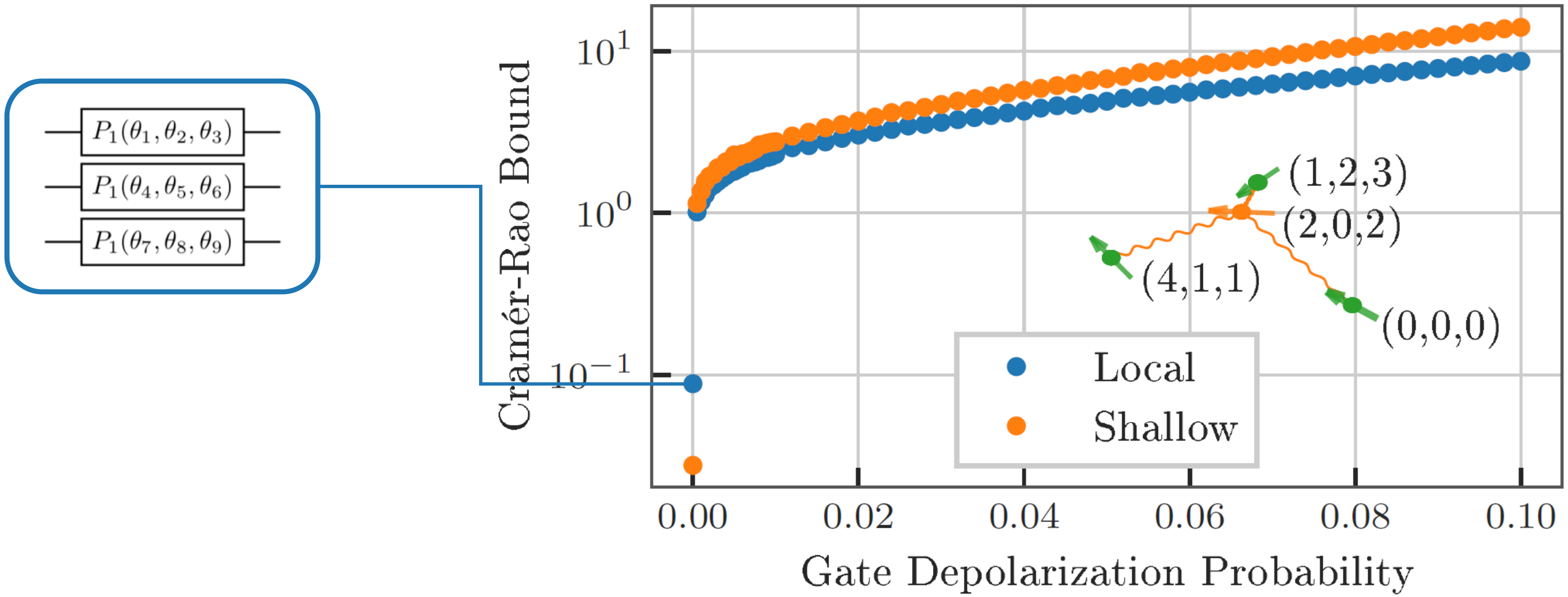

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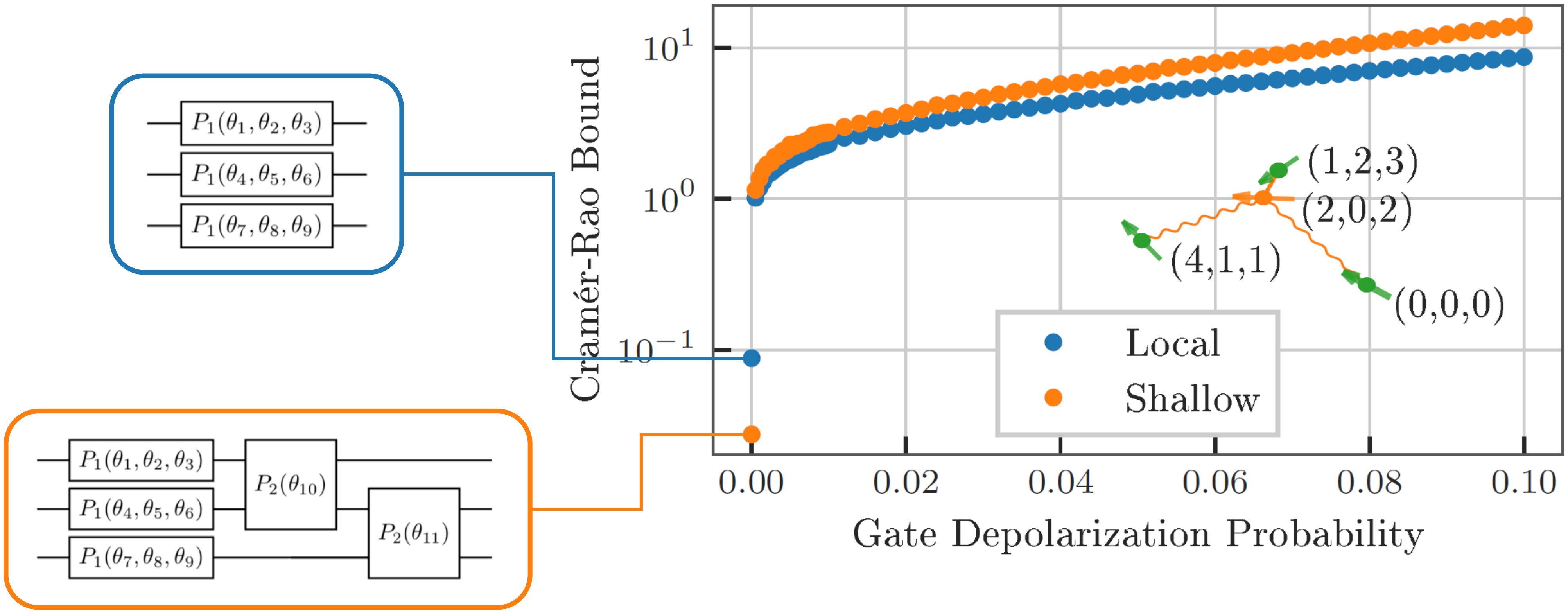

Further Contents

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We provide multiple extensions of the algorithm, for example including prior knowlege (Bayesian approach)

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We provide a parameter-shift rule for noise channels

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We provide a parameter-shift rule for noise channels

We give details on the implementation of parameter-shift rules

The Algorithm Landscape

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Single Parameter

Multiparameter

The Algorithm Landscape

Single Parameter | Multiparameter

Kaubrügger et al.

COST FUNCTION
Spin Squeezing

STATE PREPARATION
Fixed Circuit

MEASUREMENT
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The Algorithm Landscape

Single Parameter   Multiparameter

Kaubrügger et al.

COST FUNCTION
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STATE PREPARATION
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MEASUREMENT
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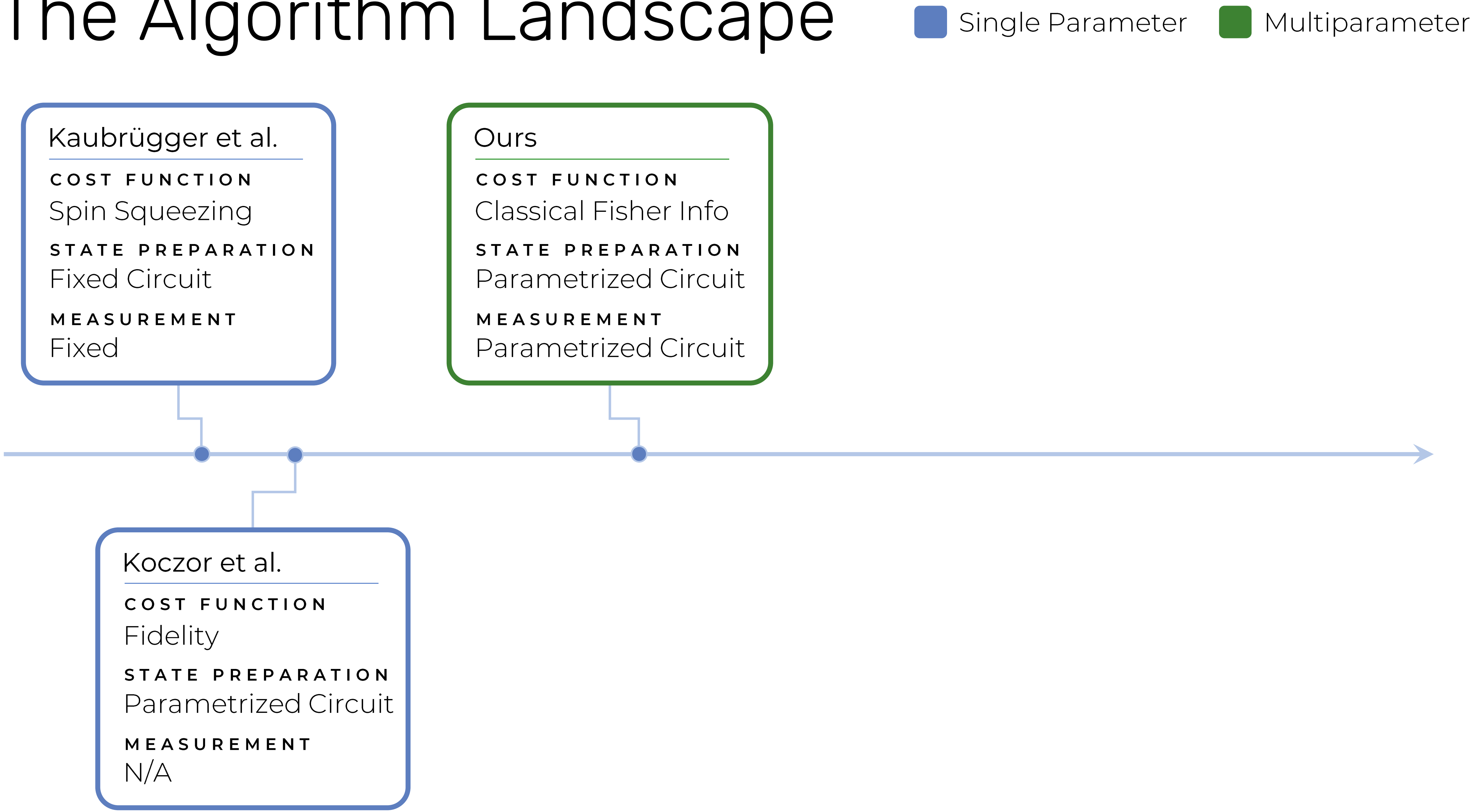
Koczor et al.

COST FUNCTION
Fidelity

STATE PREPARATION
Parametrized Circuit

MEASUREMENT
N/A

The Algorithm Landscape



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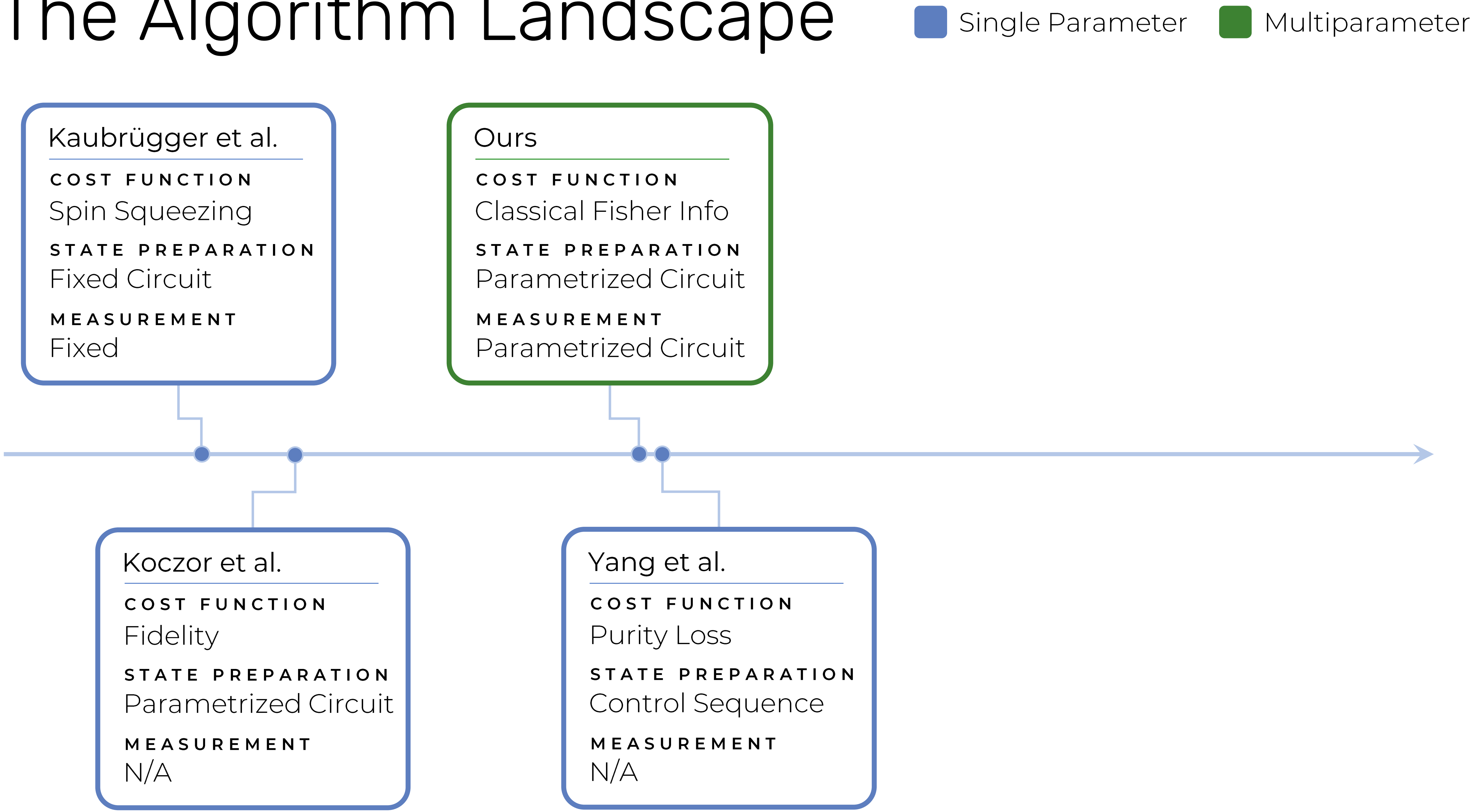

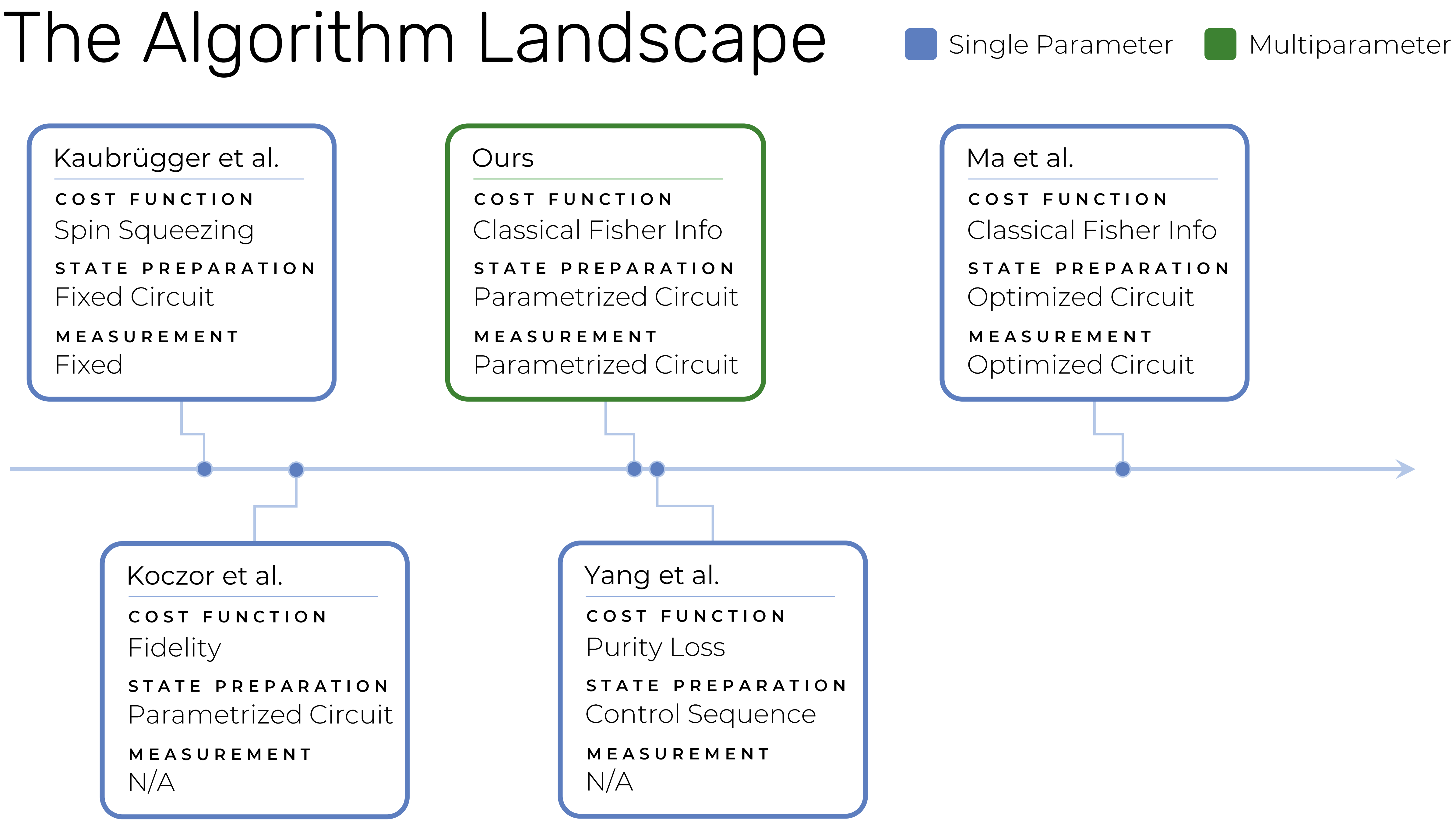
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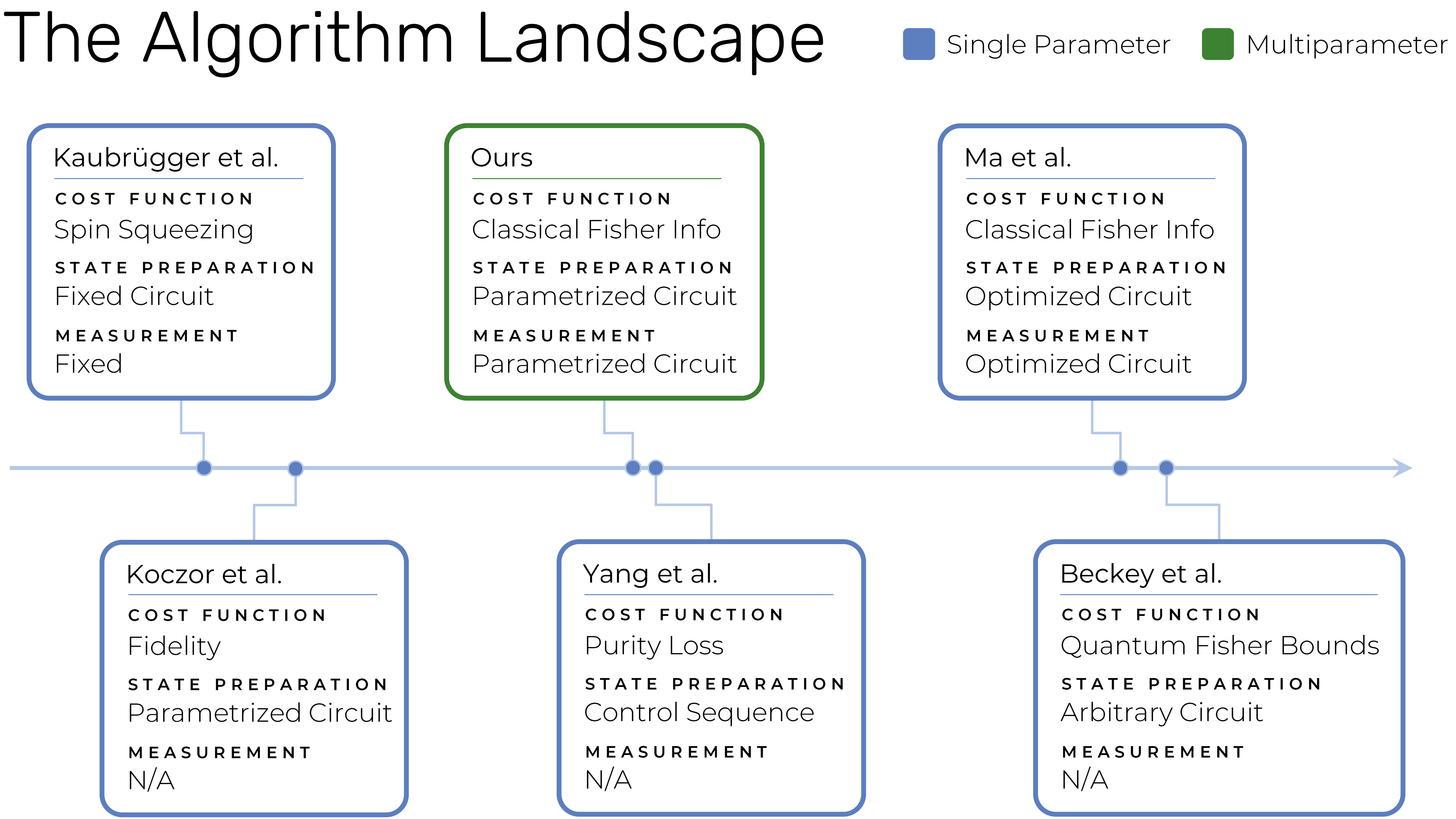

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Take-Home Message

Variational methods can be used to improve quantum sensors

Thank you for your attention!

Paper

Demo

Slides