SANZ GROUP SEMINAR

Variational Methods for Quantum Sensing and Learning

JOHANNES JAKOB MEYER, FU BERLIN & QMATH

@jj_xyz

arxiv:2006.06303

A variational toolbox for quantum multi-parameter estimation

Johannes Jakob Meyer,[1] Johannes Borregaard,[2,3] and Jens Eisert[1]

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[2]*Qutech and Kavli Institute of Nanoscience, Delft University of Technology, 2628 CJ Delft, The Netherlands*
[3]*Mathematical Sciences, Universitetsparken 5, 2100 København Ø, Matematik E, Denmark*

(Dated: June 11, 2020)

Johannes Borregaard
TU Delft

Jens Eisert
FU Berlin

Quantum Metrology

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Physical quantities (magnetic fields, energies, …)
need to be **measured** accurately

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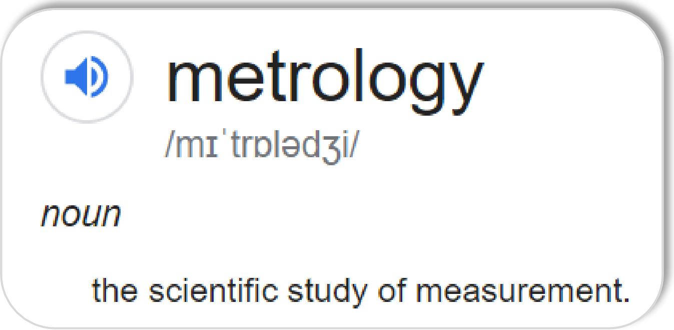

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Study how **quantum effects** can help

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noun

the scientific study of measurement.

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Probe
State

ρ

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Probe State

Encoding Evolution

$$\rho \longrightarrow \mathcal{E}(\phi)$$

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Probe State | Encoding Evolution | Measurement

$$\rho \longrightarrow \mathcal{E}(\phi) \longrightarrow \mathcal{M}$$

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Probe State		Encoding Evolution		Measurement		Output Distribution
ρ	$\longrightarrow$	$\mathcal{E}(\phi)$	$\longrightarrow$	$\mathcal{M}$	$\longrightarrow$	$p_l(\phi)$

Gathering Intuition

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$$e^{-i\phi Z}$$

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$$|+\rangle \longrightarrow e^{-i\phi Z}$$

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$$|+\rangle \longrightarrow e^{-i\phi Z} \longrightarrow |+\rangle/|-\rangle$$

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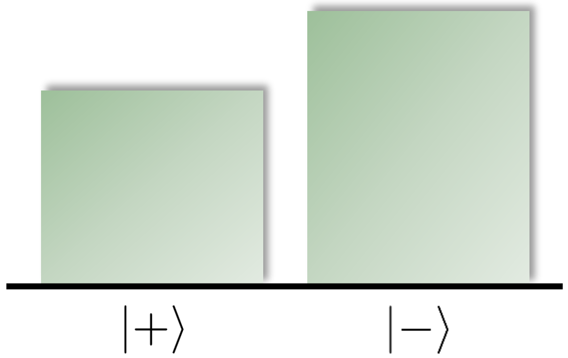

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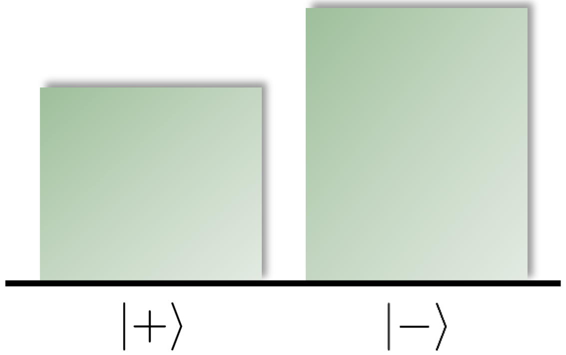

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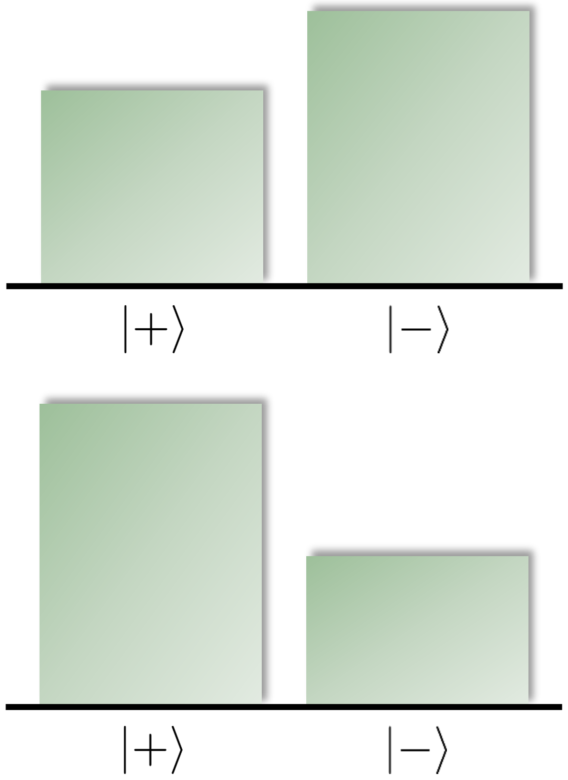

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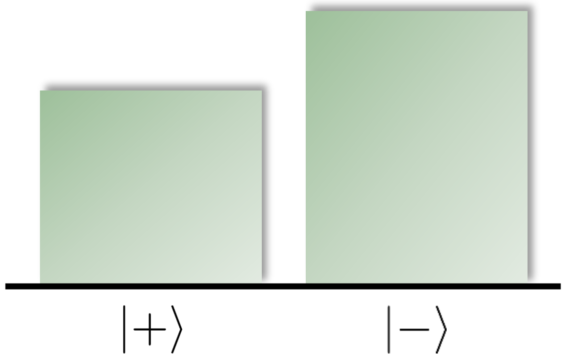

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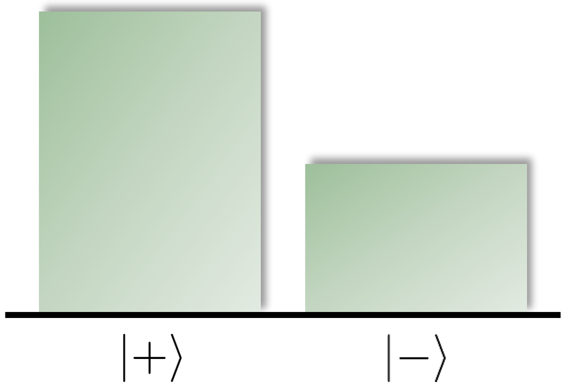

EXPERIMENT

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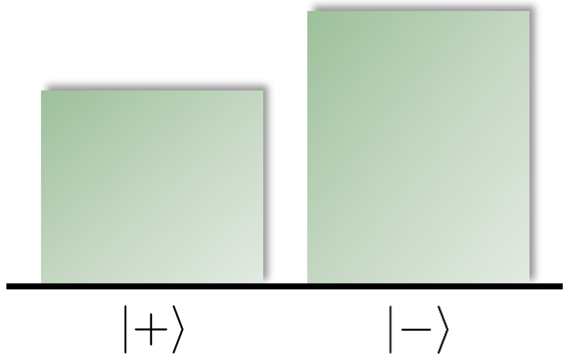

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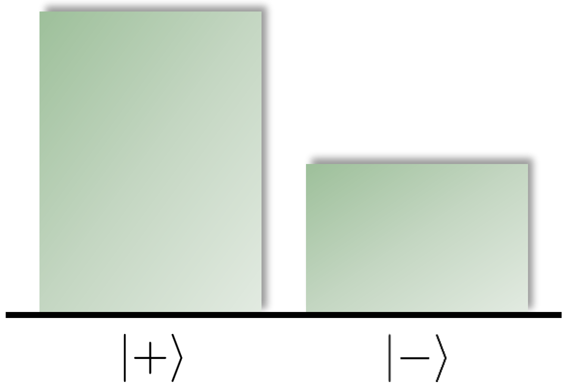

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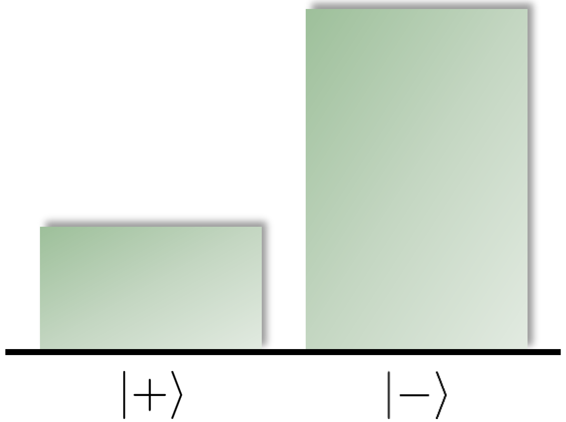

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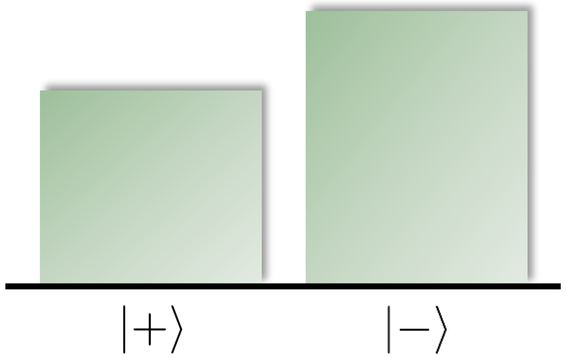

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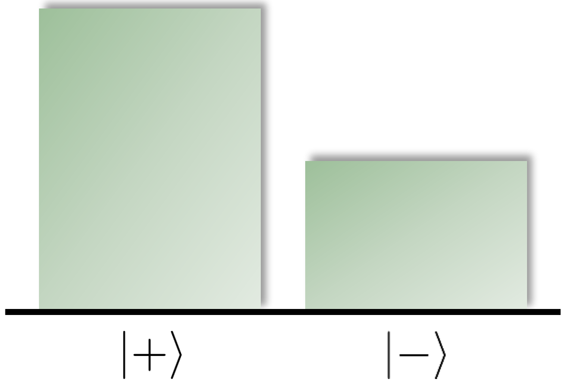

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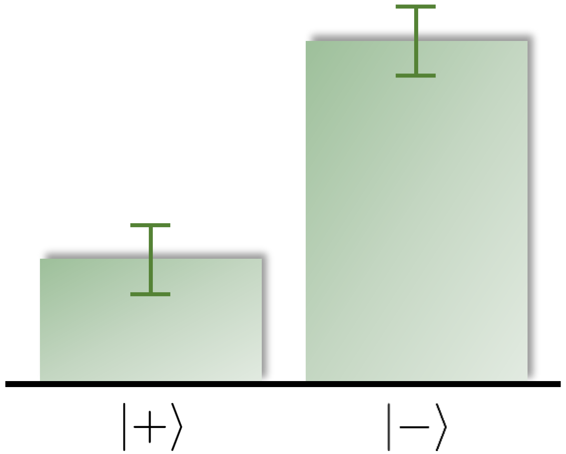

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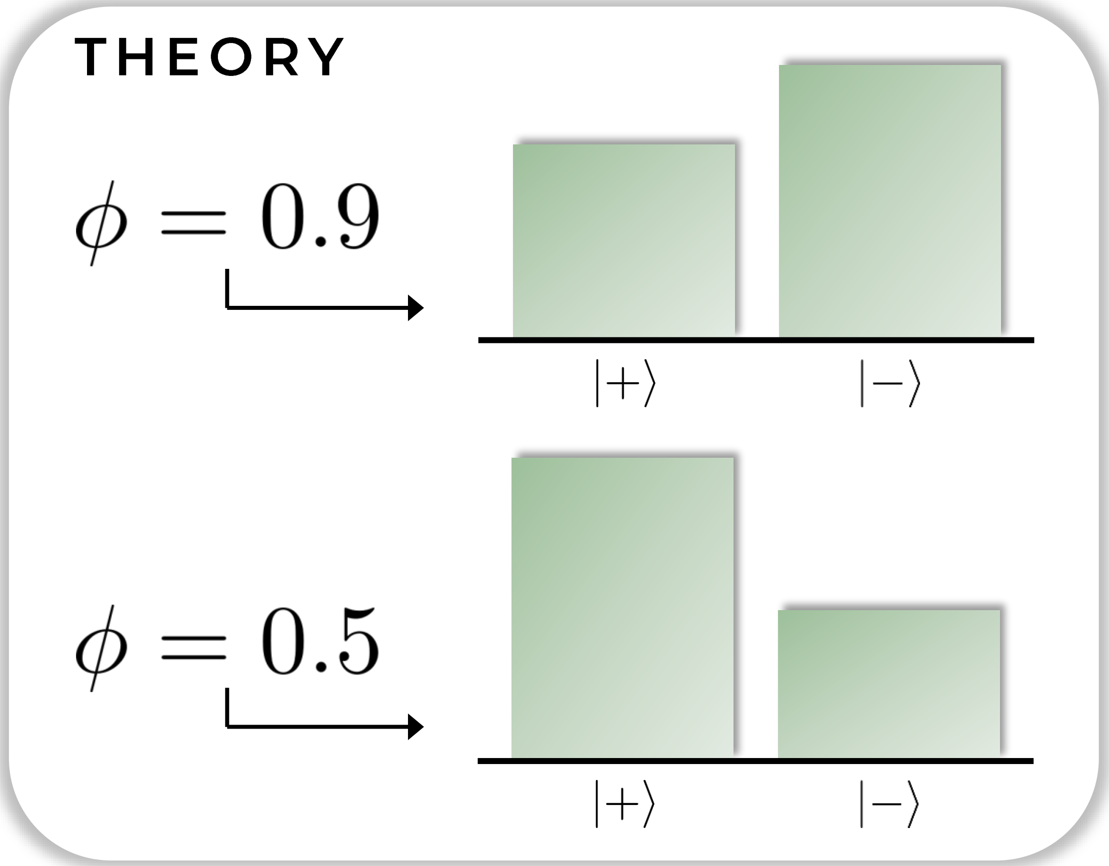

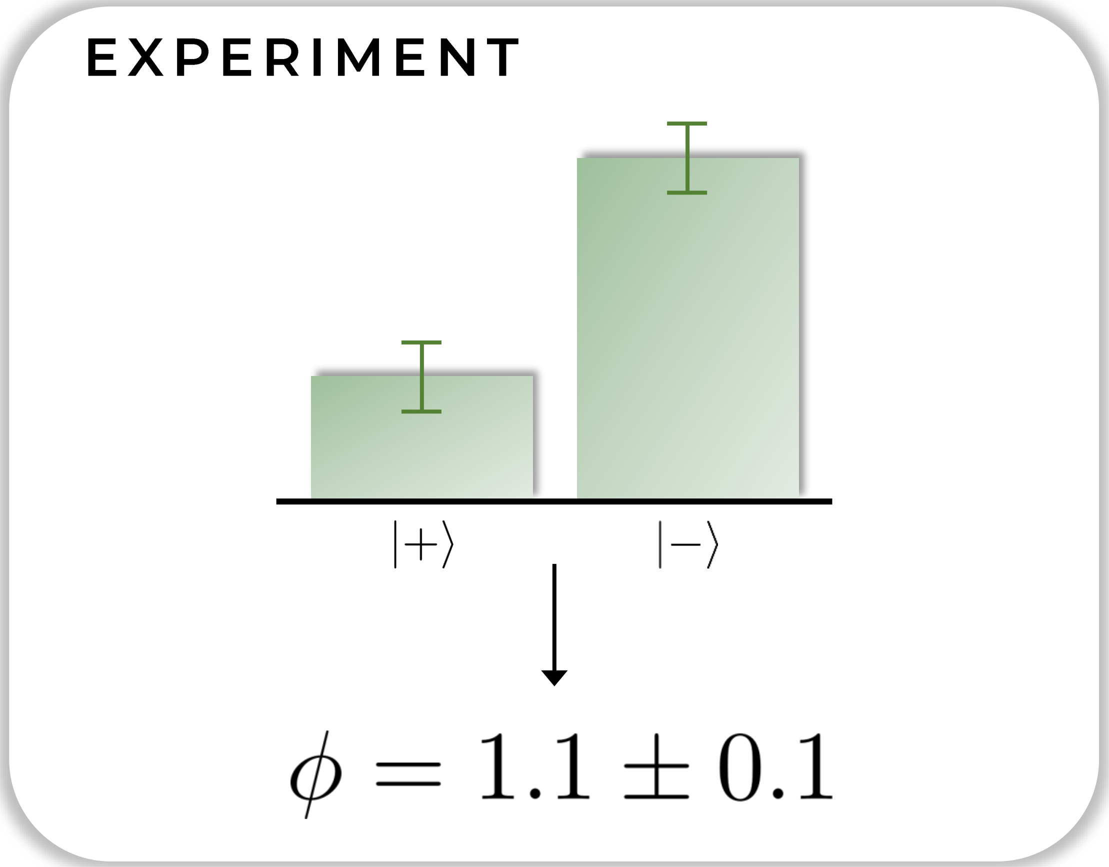

Measuring Performance

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Task: Compute an **estimator** from the output distribution

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Measuring Performance

Fisher Information in Noisy Intermediate-Scale Quantum Applications

Johannes Jakob Meyer[1,2]

[1]Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany
[2]QMATH, Department of Mathematical Sciences, Københavns Universitet, 2100 København Ø, Denmark
28-03-2021

arXiv:2103.15191

$$\mathrm{Cov}(\hat{\varphi}) \geq \frac{1}{n} I_{\phi}^{-1}(\mathcal{M}) \geq \frac{1}{n} \mathcal{F}_{\phi}^{-1}$$

Recap #1

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→ Classical Fisher information should be used to judge sensing quality!

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We need to find optimal probes and measurements

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Complicated under noise and device limitations

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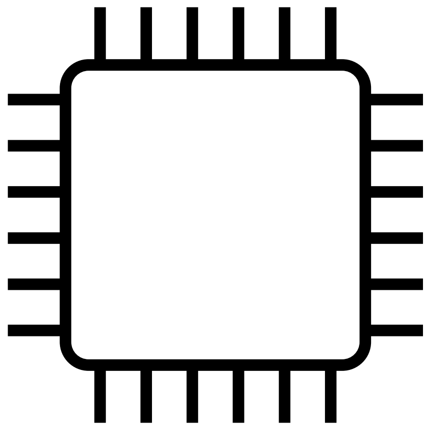

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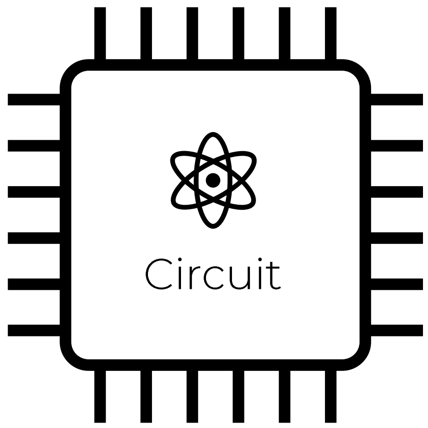

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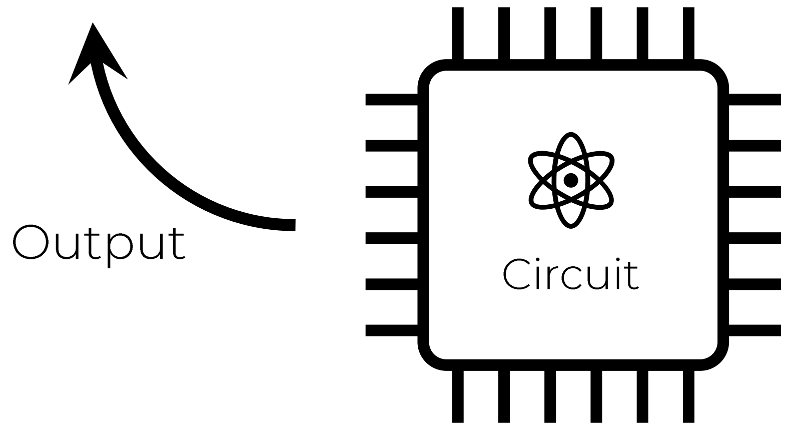

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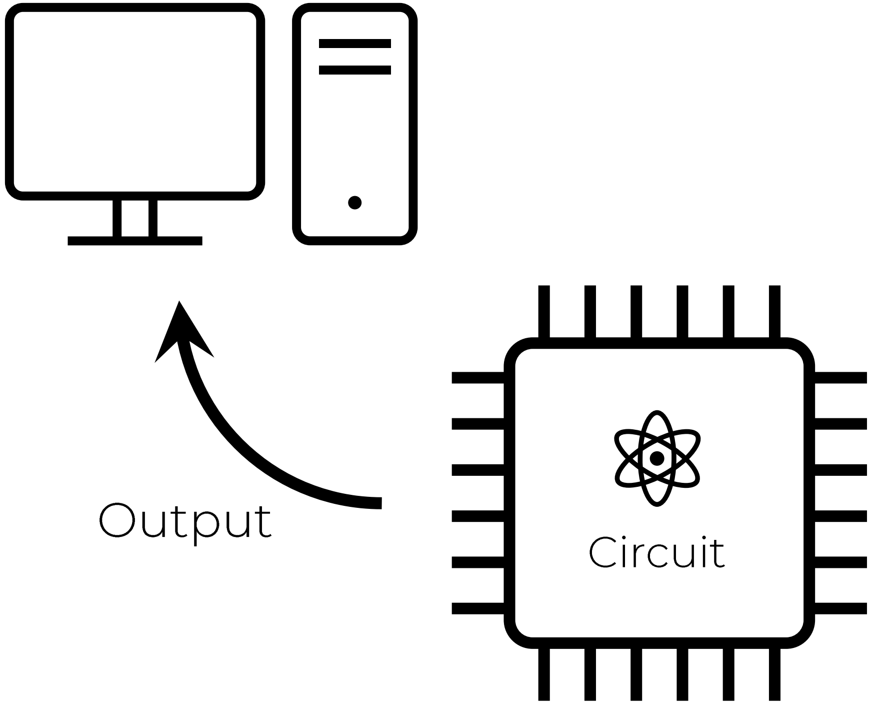

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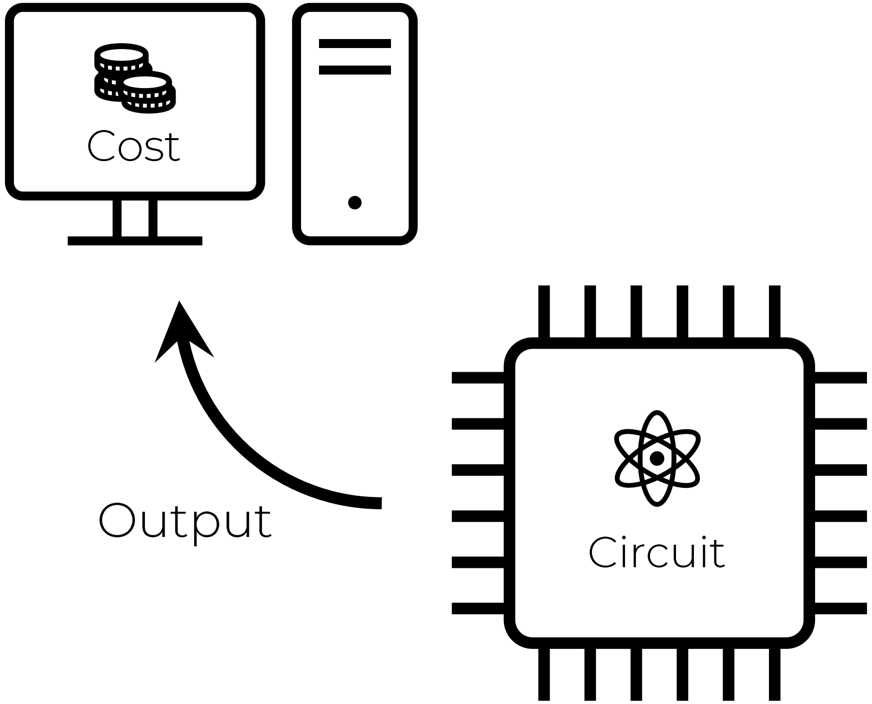

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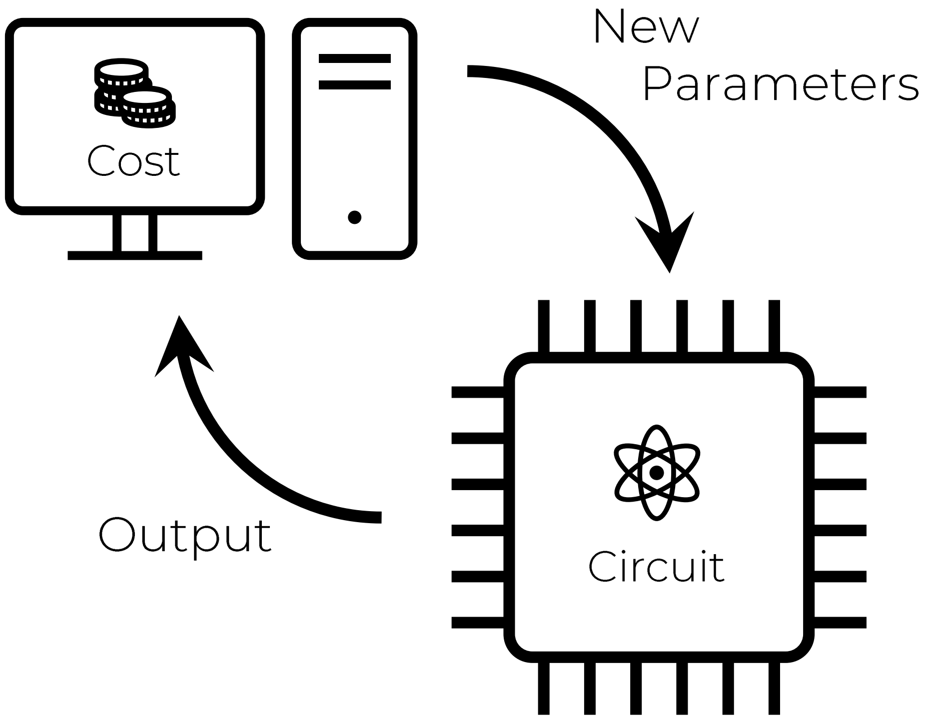

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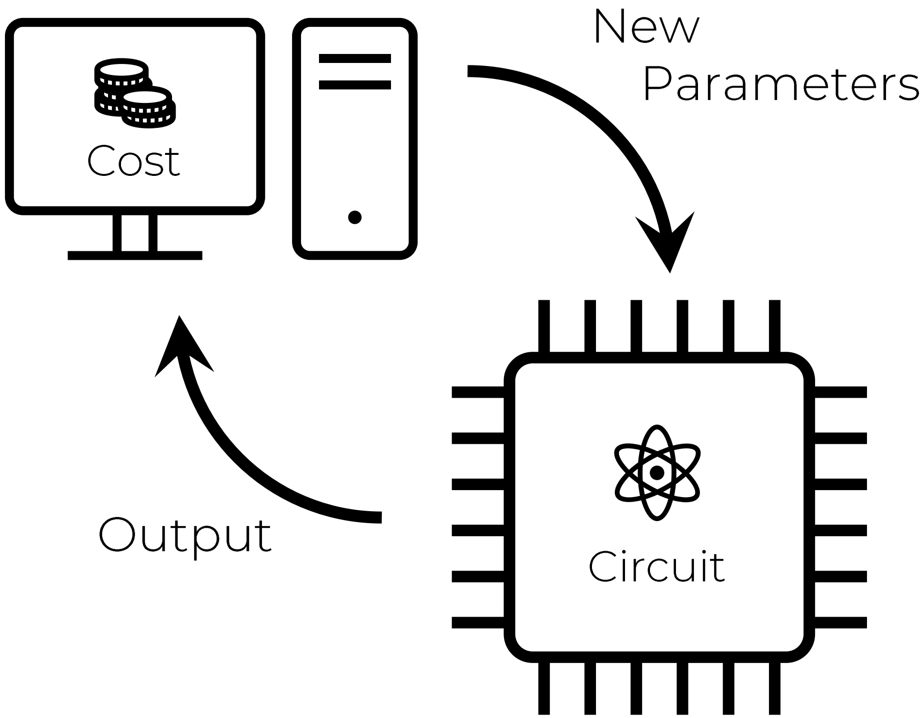

Prior work[1,2] focused on single-parameter metrology and surrogates for the Quantum Fisher Information

[1]Kaubrügger, Raphael, et al. *Physical Review Letters* 123.26 (2019): 260505.
[2]Koczor, Bálint, et al. "Variational-State Quantum Metrology." *New Journal of Physics* (2020).

Cost Function

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Include post-processing by considering a function of the parameters:
Exploit transformation rule of Fisher Information Matrix

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Need a scalar cost function:
Apply weighted trace to both sides of the CRB!

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$$\mathrm{Tr}\{W\,\mathrm{Cov}(\hat{\boldsymbol{f}})\} \geq \frac{1}{n}\,\mathrm{Tr}\{W I_{\boldsymbol{f}}^{-1}\} = \frac{1}{n} C_W$$

Calculation of Fisher Information

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Fisher Information Matrix w.r.t. the physical parameters

$$[I_{\boldsymbol{\phi}}]_{jk} = \sum_l \frac{1}{p_l} \frac{\partial p_l}{\partial \phi_j} \frac{\partial p_l}{\partial \phi_k}$$

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Exploit parameter-shift rule[1,2] to calculate derivatives

$$\partial_j p_l(\boldsymbol{\phi}) = \frac{1}{2} \left[p_l \left(\boldsymbol{\phi} + \frac{\pi}{2} \boldsymbol{e}_j \right) - p_l \left(\boldsymbol{\phi} - \frac{\pi}{2} \boldsymbol{e}_j \right) \right]$$

[1] Schuld, Maria, et al. *Physical Review A* 99.3 (2019): 032331.
[2] Banchi, Leonardo, and Gavin E. Crooks. arXiv preprint arXiv:2005.10299 (2020).

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The cost function is obtained from a weighted trace of the Cramér-Rao bound

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Numerical experiments that showcase the performance of the approach

The Algorithm Landscape

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Single Parameter

Multiparameter

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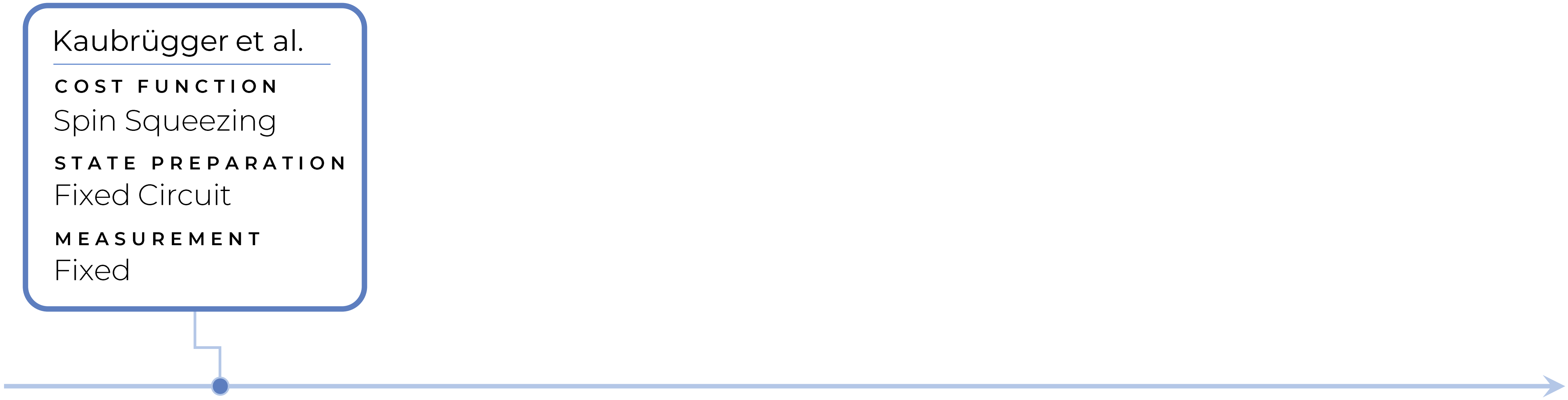


The Algorithm Landscape

Single Parameter   Multiparameter

Kaubrügger et al.

COST FUNCTION
Spin Squeezing

STATE PREPARATION
Fixed Circuit

MEASUREMENT
Fixed

Koczor et al.

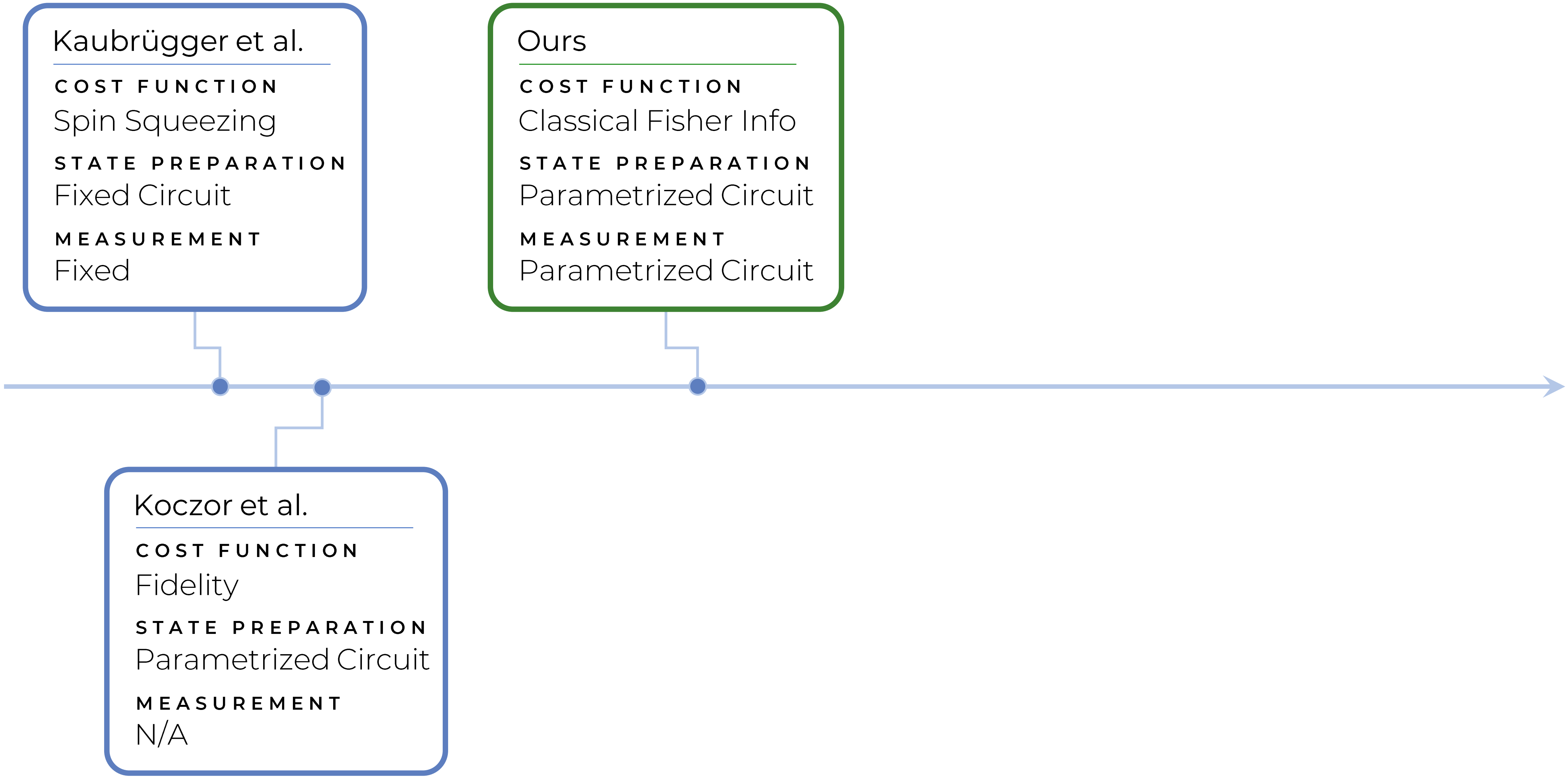
COST FUNCTION
Fidelity

STATE PREPARATION
Parametrized Circuit

MEASUREMENT
N/A

The Algorithm Landscape

Single Parameter Multiparameter



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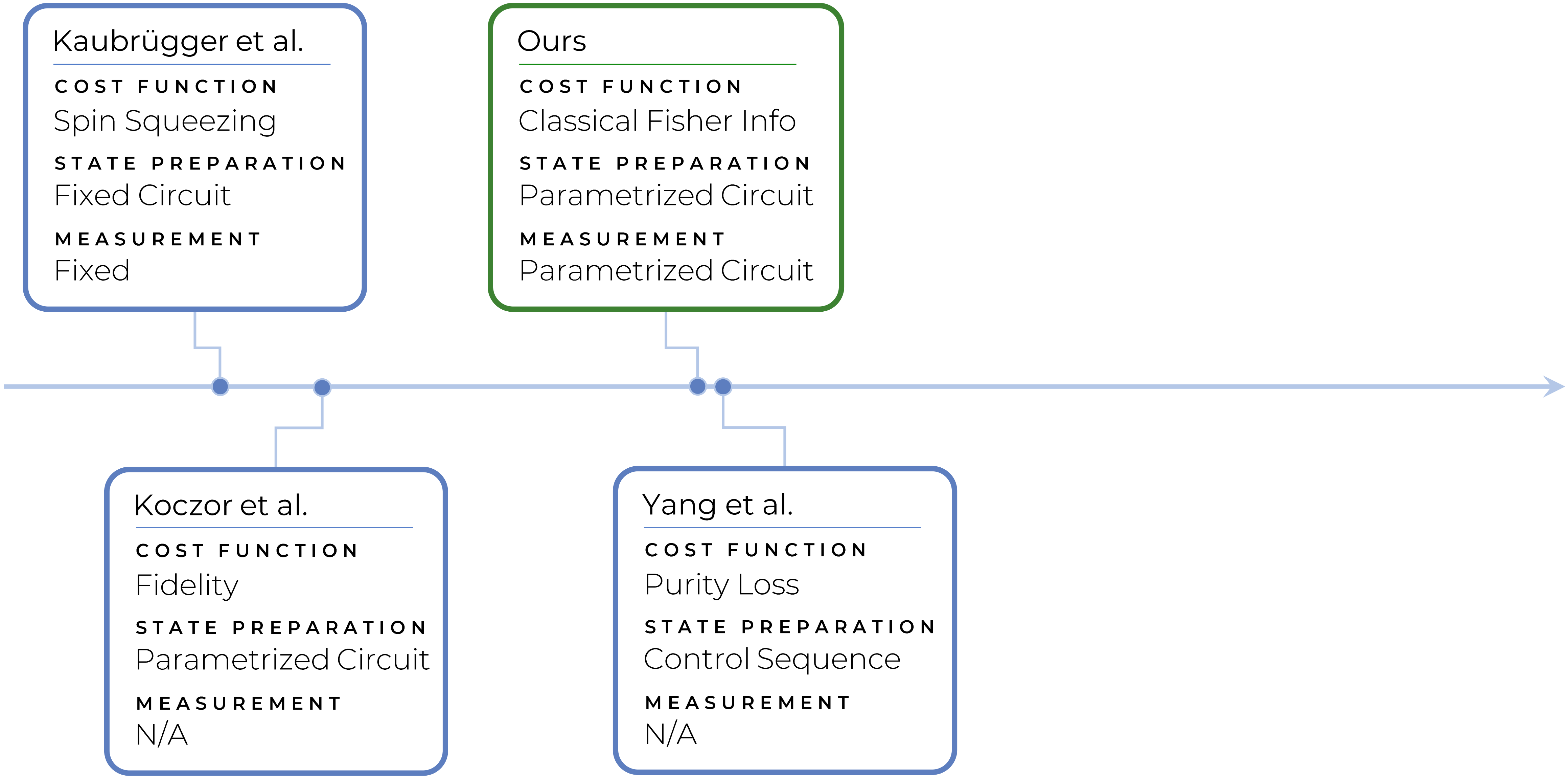

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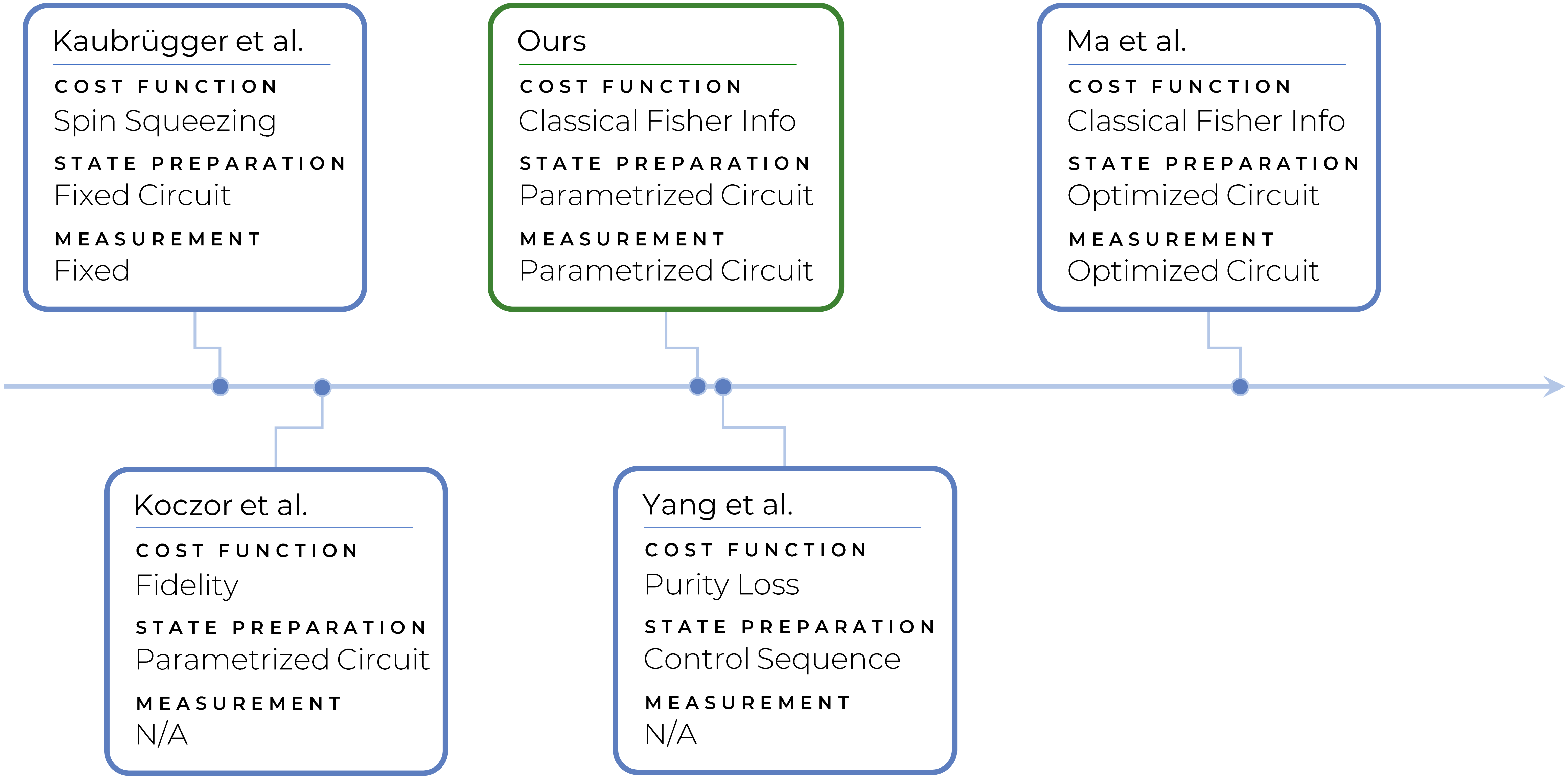

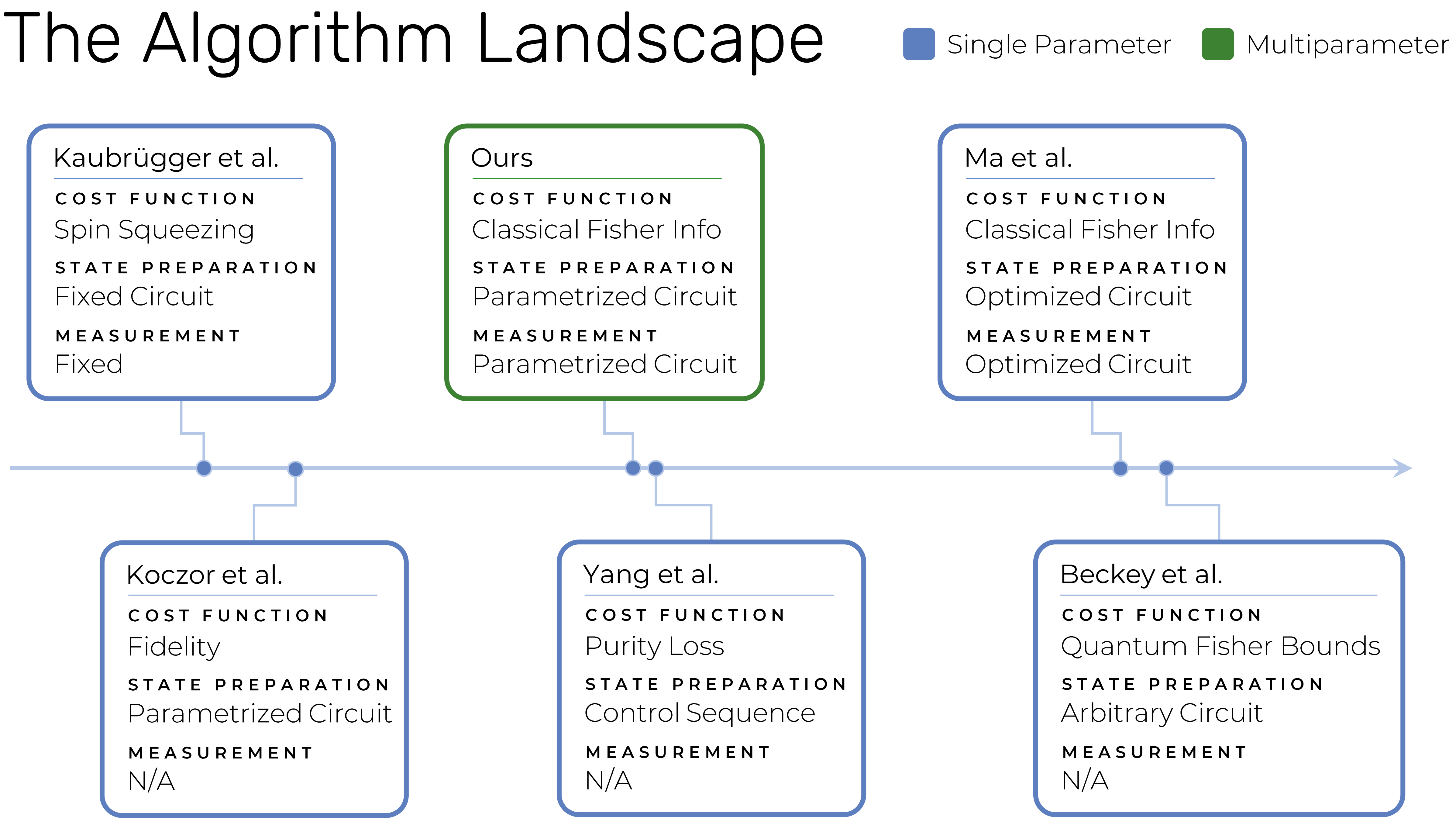
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N/A
Yang et al.
COST FUNCTION
Purity Loss
STATE PREPARATION
Control Sequence
MEASUREMENT
N/A
Beckey et al.
COST FUNCTION
Quantum Fisher Bounds
STATE PREPARATION
Arbitrary Circuit
MEASUREMENT
N/A

Take-Home Message #1

> Variational methods on near-term quantum computers can be used to improve quantum sensors

Paper

Demo

Slides

Fisher Note

10.1103/PhysRevA.103.032430

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The effect of data encoding on the expressive power of variational quantum machine learning models

Maria Schuld,[1] Ryan Sweke,[2] and Johannes Jakob Meyer[2]

[1] *Xanadu, Toronto, ON, M5G 2C8, Canada*
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(Dated: August 21, 2020)

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Disclaimer: After uploading to the arxiv we were notified of work by Vidal and Theis[1] that has significant overlap with ours.

[1]Gil Vidal, Francisco Javier, and Dirk Oliver Theis. "Input Redundancy for Parameterized Quantum Circuits." *Frontiers in Physics* 8 (2020): 297.

Maria Schuld
Xanadu

Ryan Sweke
FU Berlin

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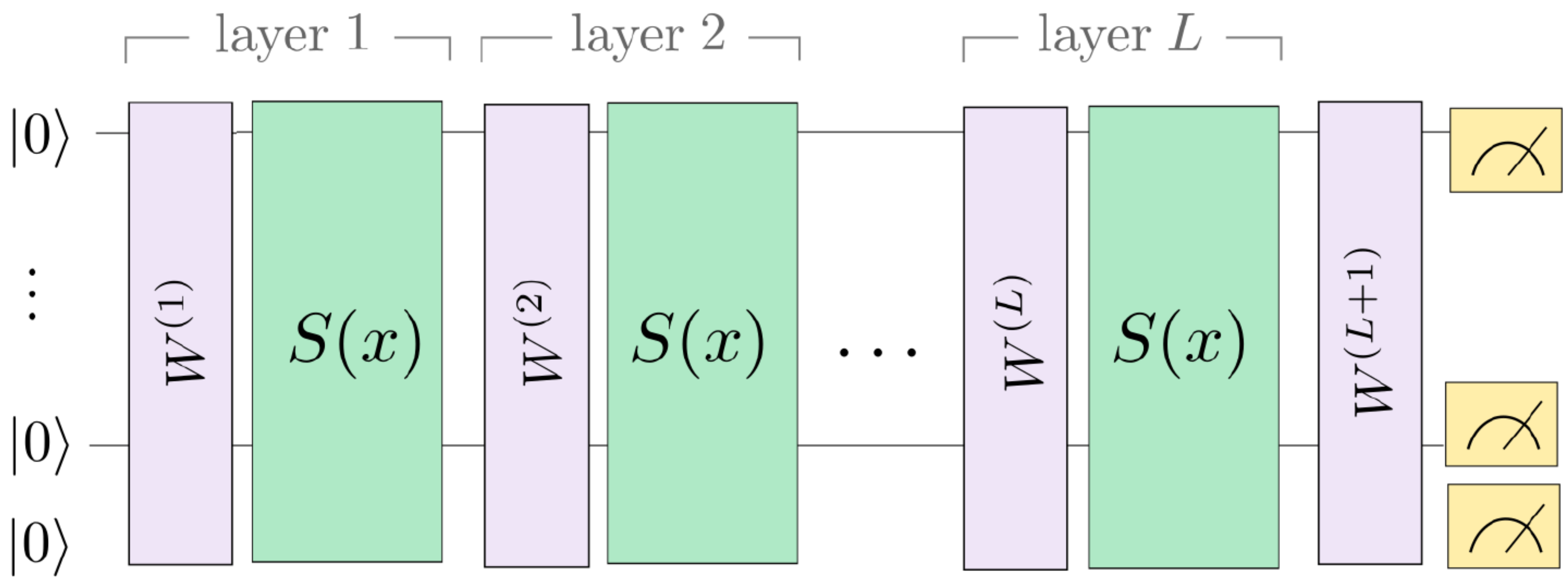
A lot of work has been done to understand the practical side, but very little is known on the theory side

So we asked ourselves: **What functions can such models learn?**

Variational Quantum Learning Models



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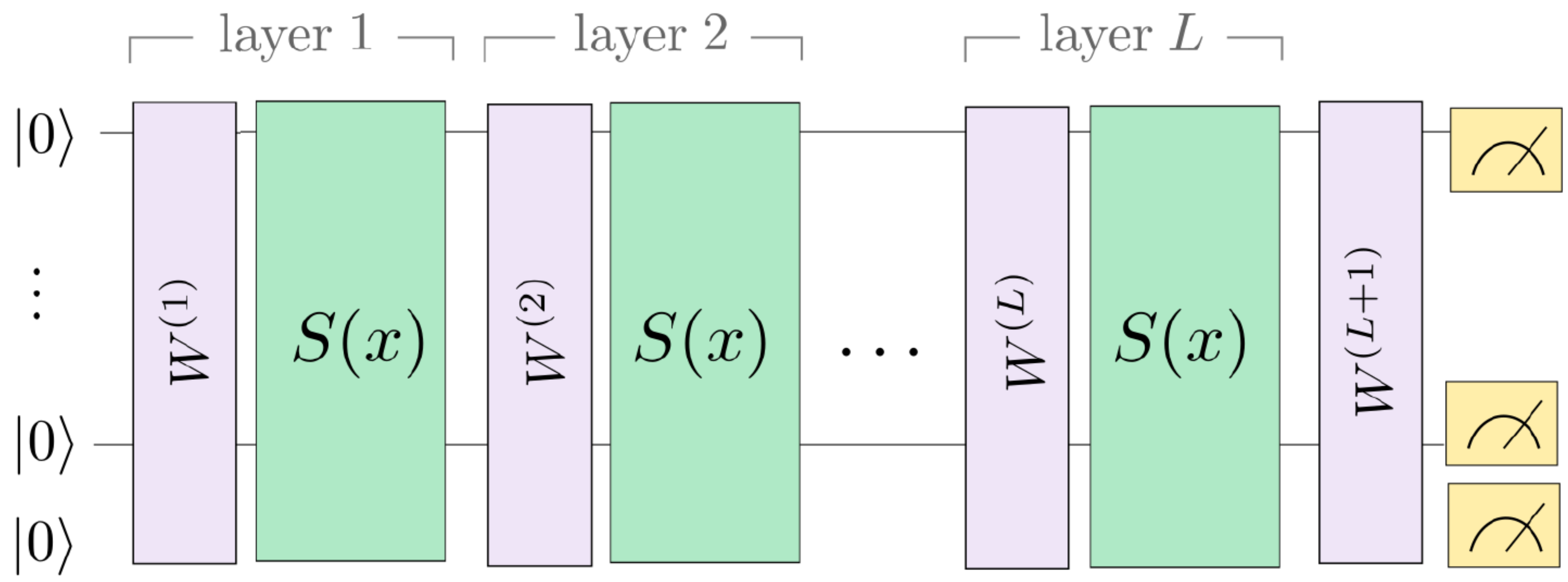
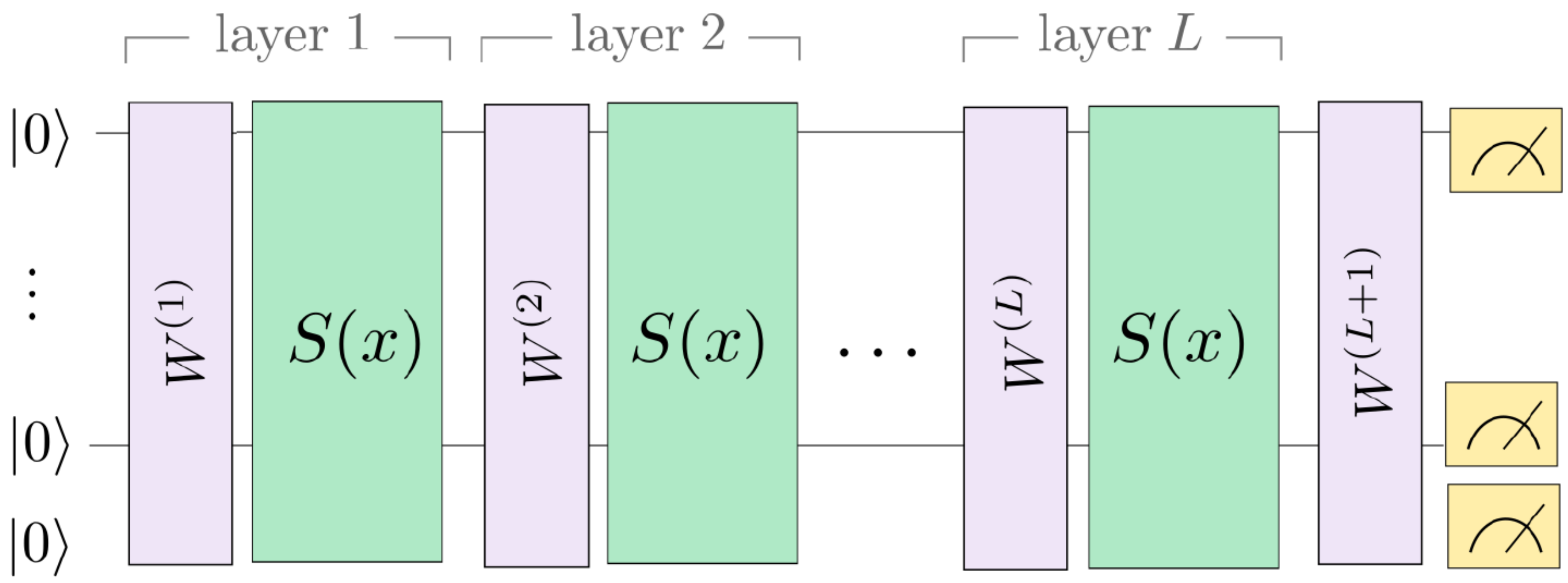


$$f(x) = \langle M \rangle$$

Model Output

Variational Quantum Learning Models



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Model Output

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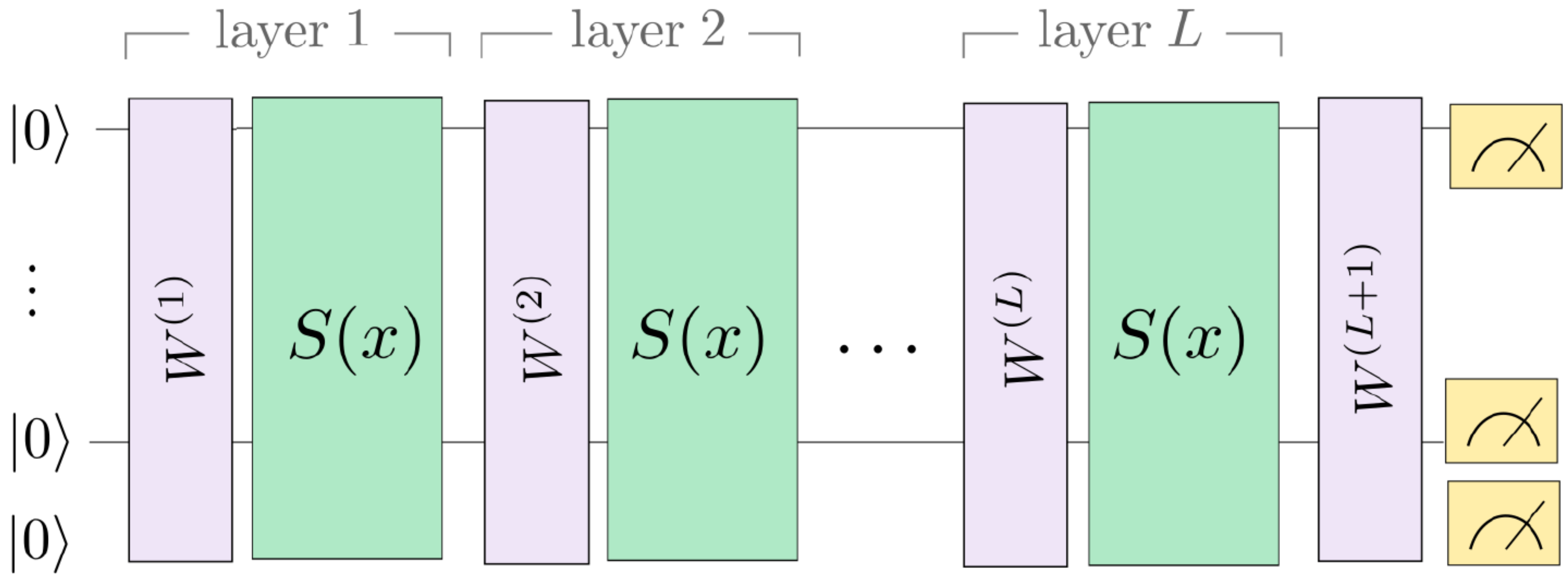
Hamiltonian Evolution

Variational Quantum Learning Models



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Hamiltonian Evolution

$$W^{(l)}(\boldsymbol{\theta})$$

Trainable Blocks

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$$|\psi\rangle = \sum_{\lambda} \psi_{\lambda}|\lambda\rangle$$

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and the frequencies accumulate between layers:

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The output state contains all possible sums of frequencies

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Output is expectation value and therefore contains a complex conjugation

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The accessible spectrum consists of all sums of differences of eigenvalues of the generator of the data encoding

Take-Home Message #2

Quantum learning models output Fourier series, repeating data encoding gives access to higher frequencies

Pauli-Encodings

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Pauli rotations are the most popular encoding strategy, e.g.

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The number of available frequencies grows linearly in depth and width

But for general encodings the dependence can be exponential:

$$\#(\text{frequencies}) \leq \frac{d^{2L}}{2} - 1$$

d System dimension

L Number of layers

Consequences for Learning Tasks

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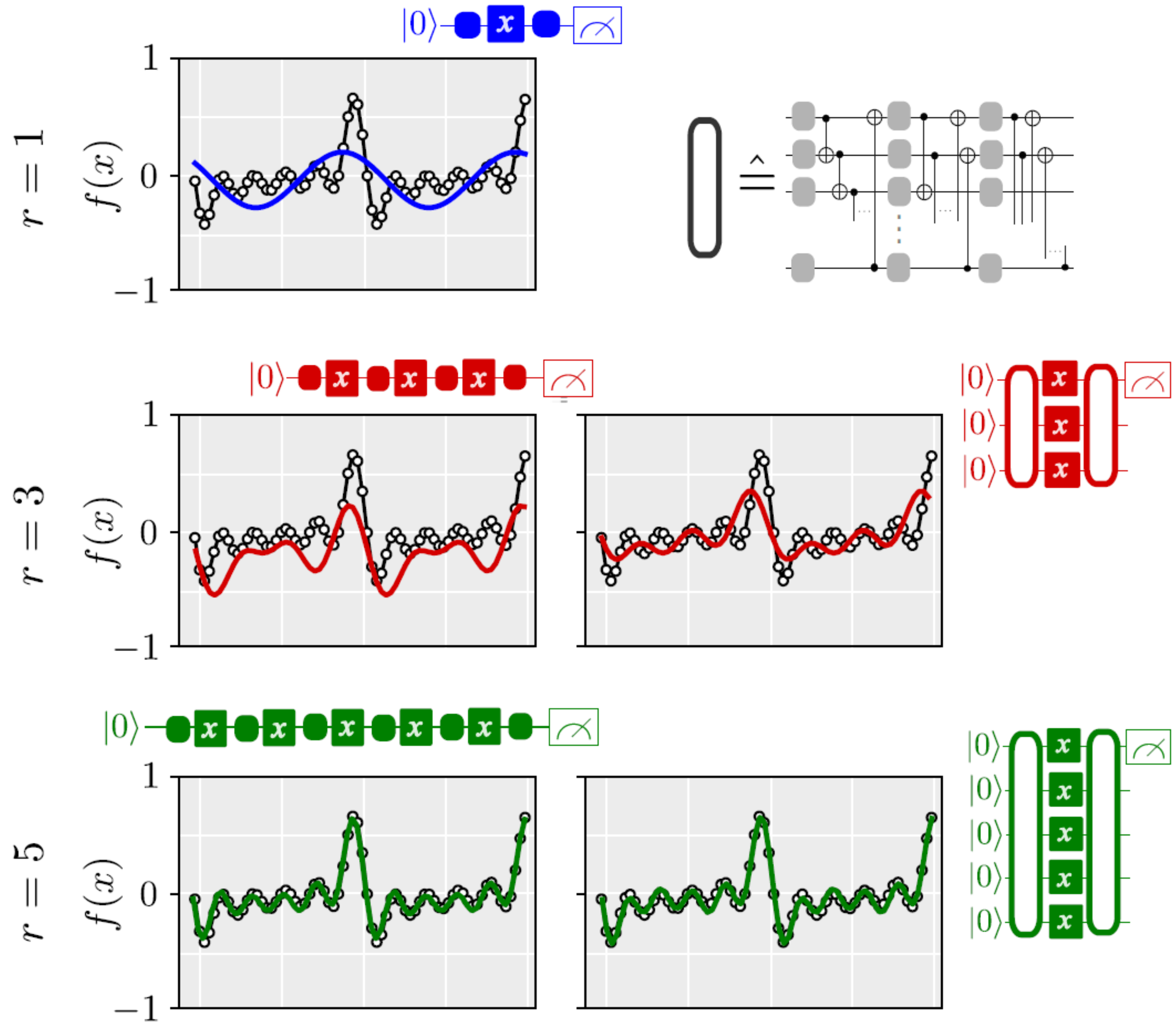
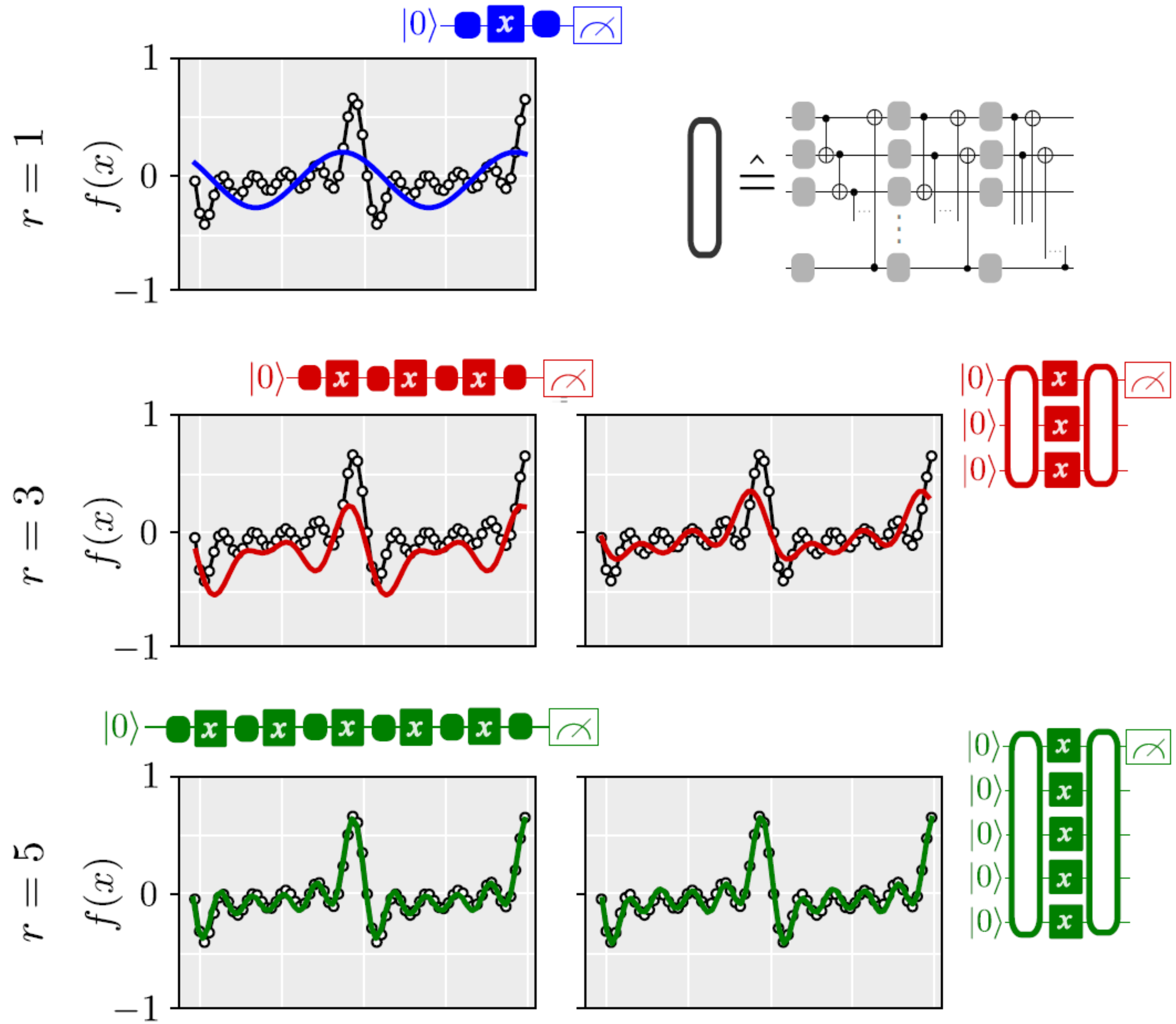


Consequences for Learning Tasks



You can reproduce all figures from the paper at home!

Distribution of Fourier Coefficients

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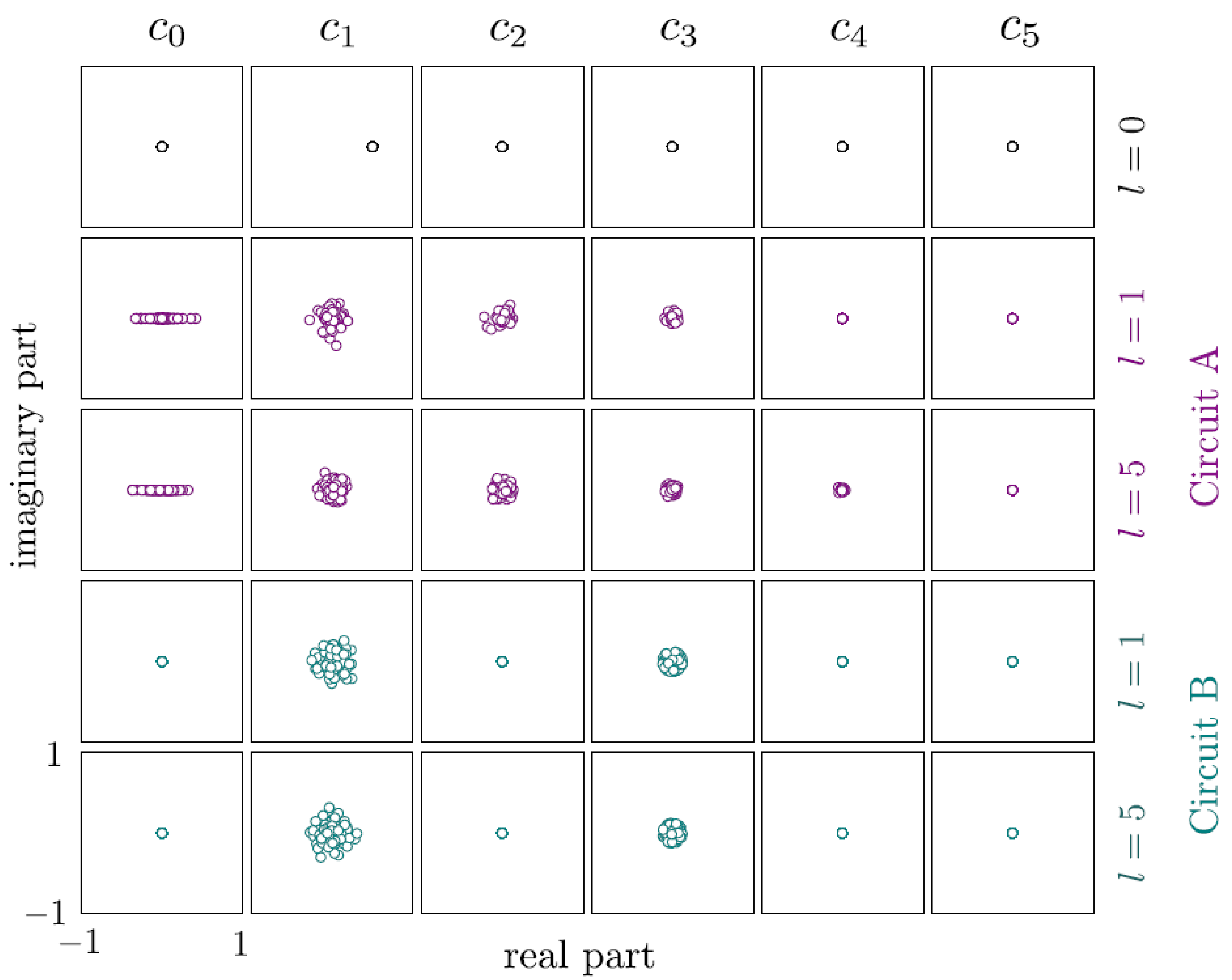

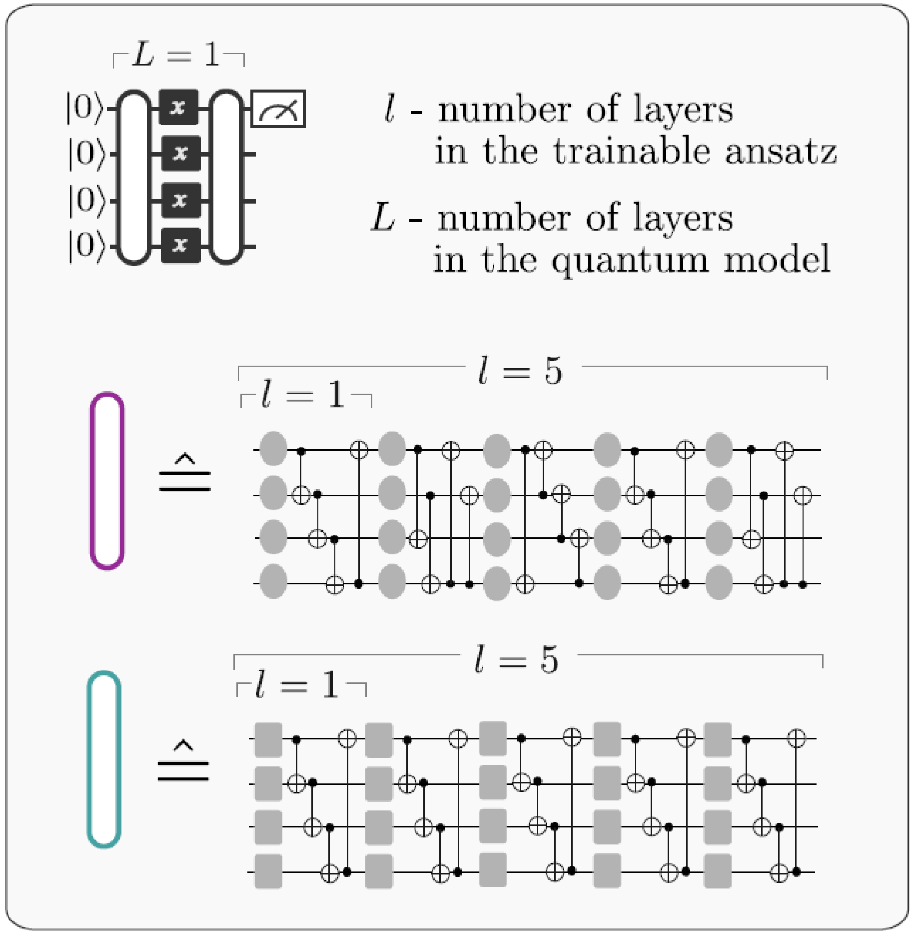

Universality of Quantum Models

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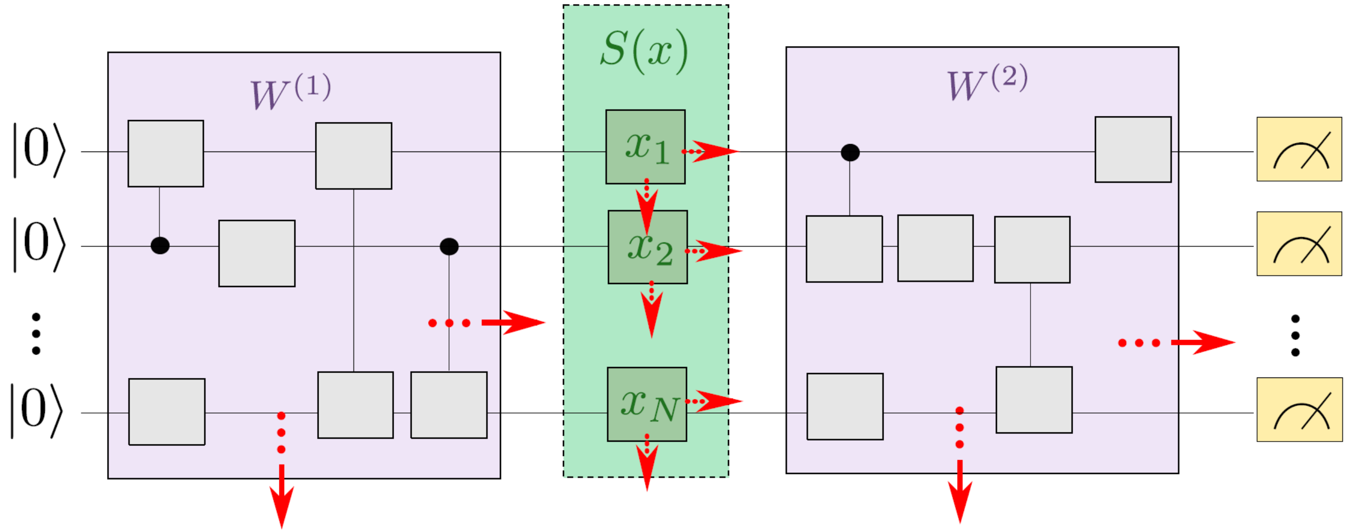
A model with one layer of data encodings generated by a *universal Hamiltonian family* and arbitrary unitaries is a universal function approximator



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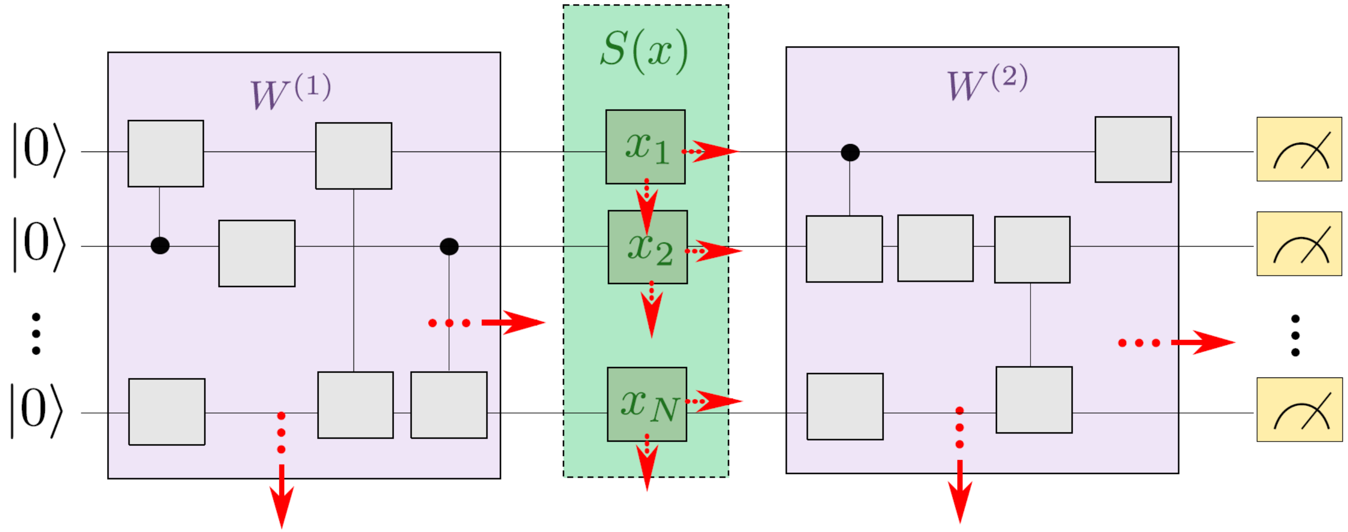

A universal Hamiltonian family asymptotically has access to all integer frequencies. Repeated single-qubit Pauli rotation encodings are a universal Hamiltonian family!

Take-Home Message #3

> Quantum learning models are universal function approximators

Practical Implications for Model Building

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5. Make your observables trainable

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4. Are quantum models good for signal processing?

Thank you for your attention!

Paper

Demo

Slides