Q-TURN 2020

Improving Quantum Sensing with Quantum Computers

JOHANNES JAKOB MEYER, FU BERLIN

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arxiv:2006.06303

A variational toolbox for quantum multi-parameter estimation

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[2]*Qutech and Kavli Institute of Nanoscience, Delft University of Technology, 2628 CJ Delft, The Netherlands*
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(Dated: June 11, 2020)

Johannes Borregaard
TU Delft

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Quantum Metrology

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Physical quantities (magnetic fields, energies, ...)
need to be **measured** accurately

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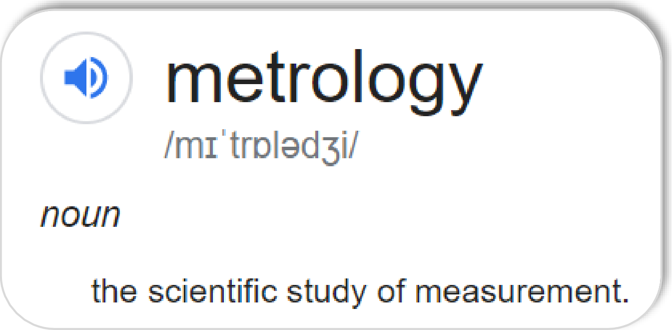

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Study how **quantum effects** can help

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noun

the scientific study of measurement.

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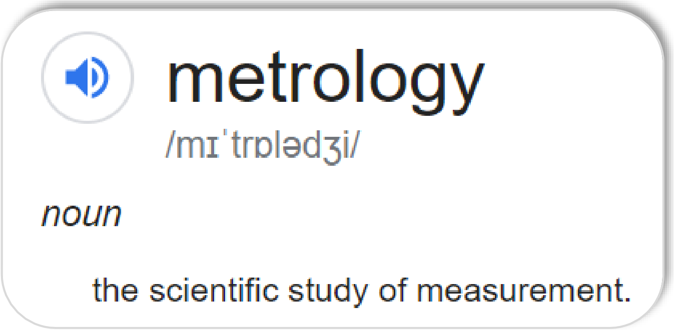

ρ

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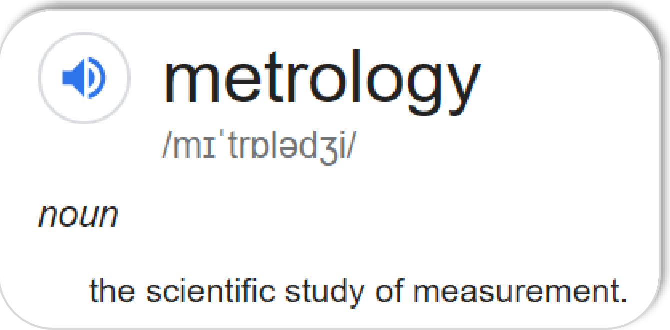

Probe State

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Probe State		Encoding Evolution
ρ	$\longrightarrow$	$\mathcal{E}(\phi)$

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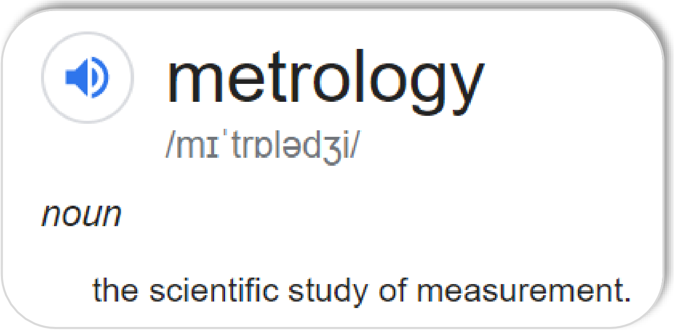

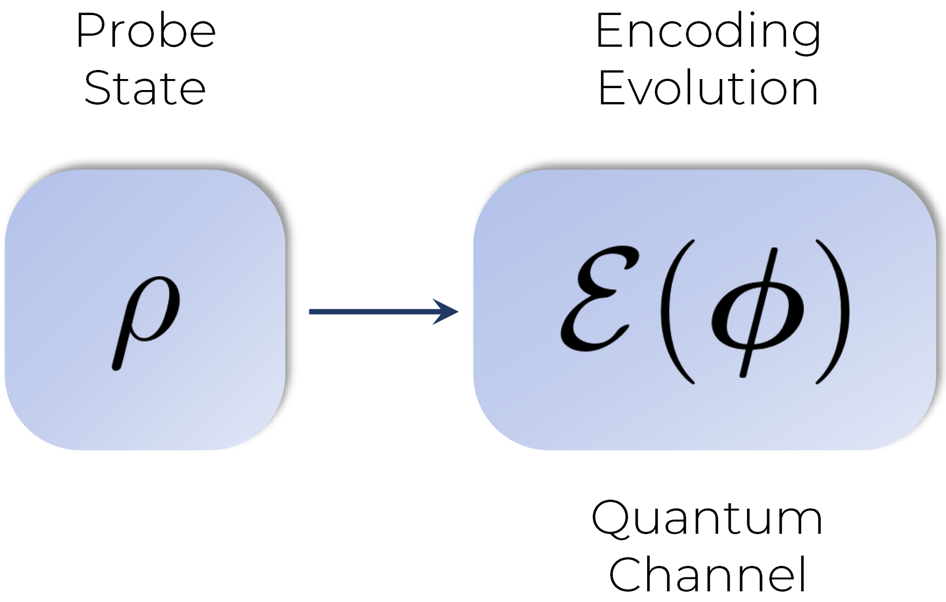

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Probe State	Encoding Evolution	Measurement
ρ	$\mathcal{E}(\phi)$	$\{\Pi_l\}$
	Quantum Channel	

$$\rho \longrightarrow \mathcal{E}(\phi) \longrightarrow \{\Pi_l\}$$

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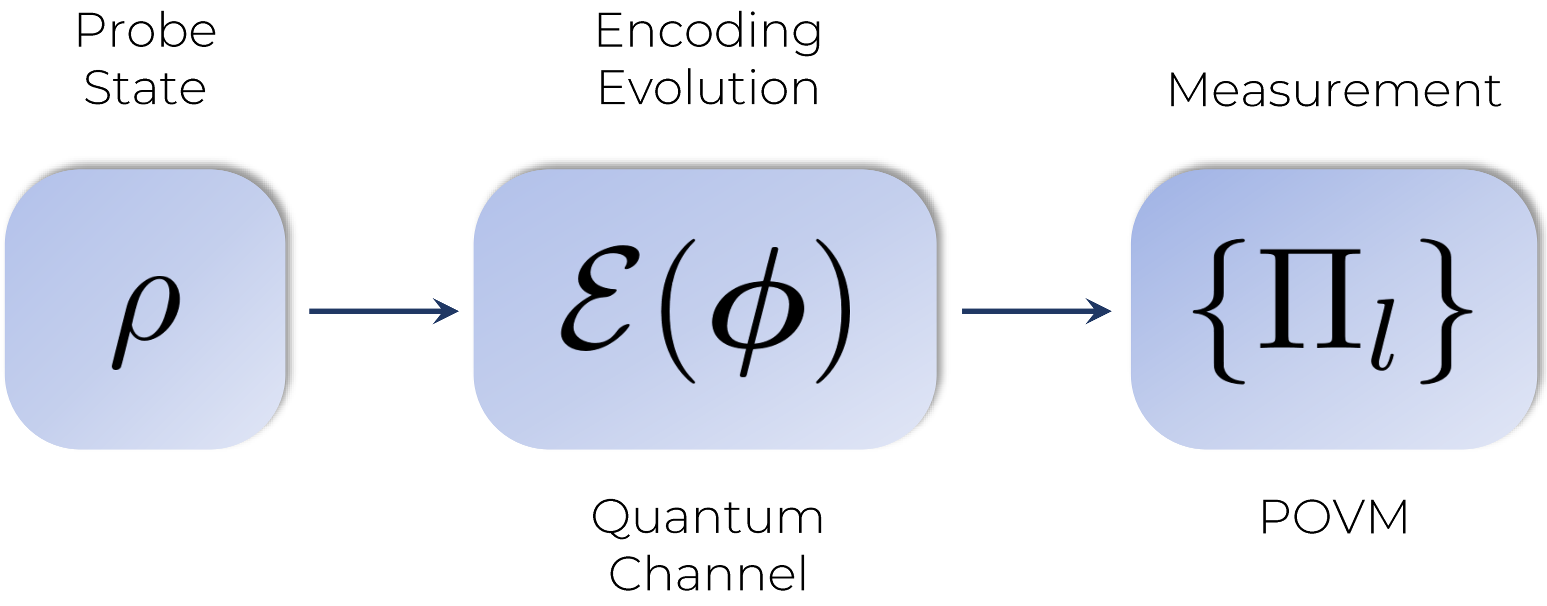

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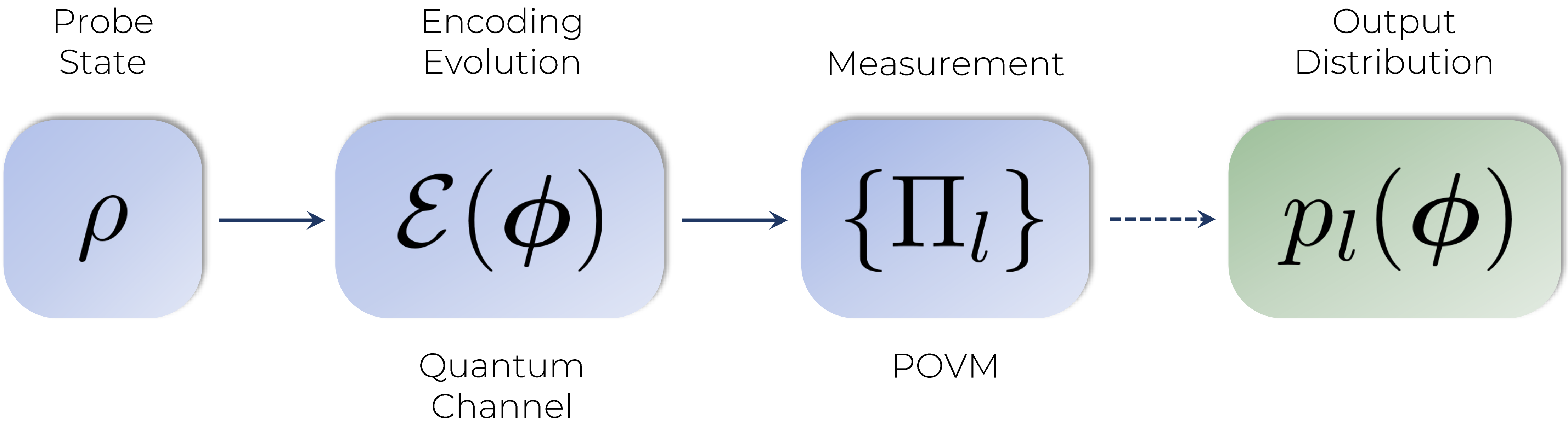

Cramér-Rao Bound

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Task: Compute an **estimator** from the output distribution

$$p_l(\boldsymbol{\phi}) \;\rightarrow\; \hat{\boldsymbol{\varphi}} \colon \mathbb{E}\{\hat{\boldsymbol{\varphi}}\} = \boldsymbol{\phi}$$

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Sensing amounts to estimating the underlying physical parameters from a classical probability distribution

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→ Classical Fisher information should be used to judge sensing quality!

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We need to find optimal probes and measurements

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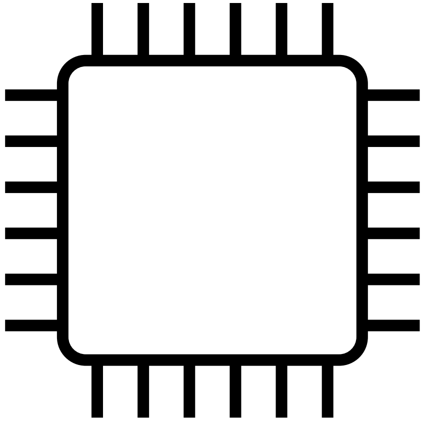

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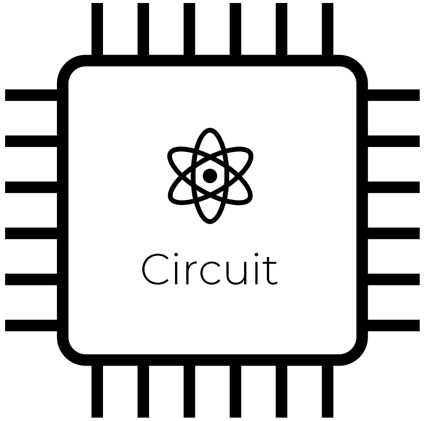

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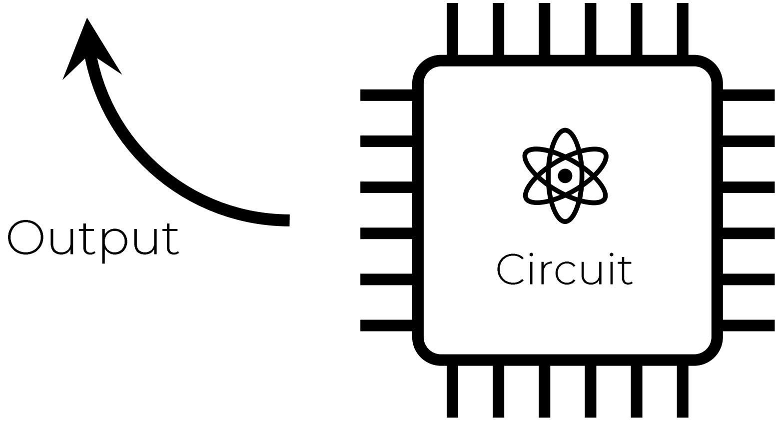

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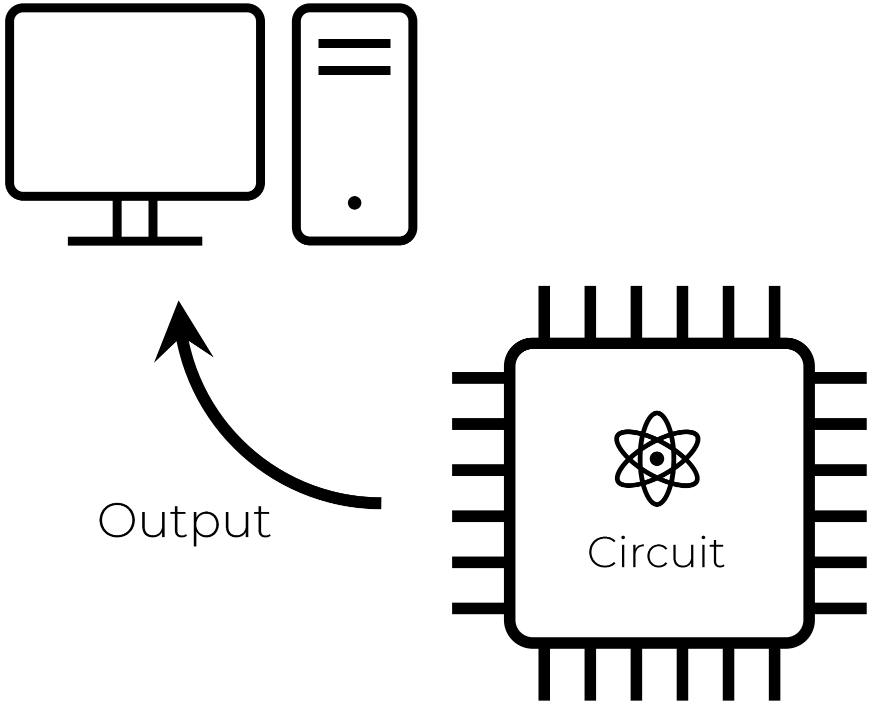

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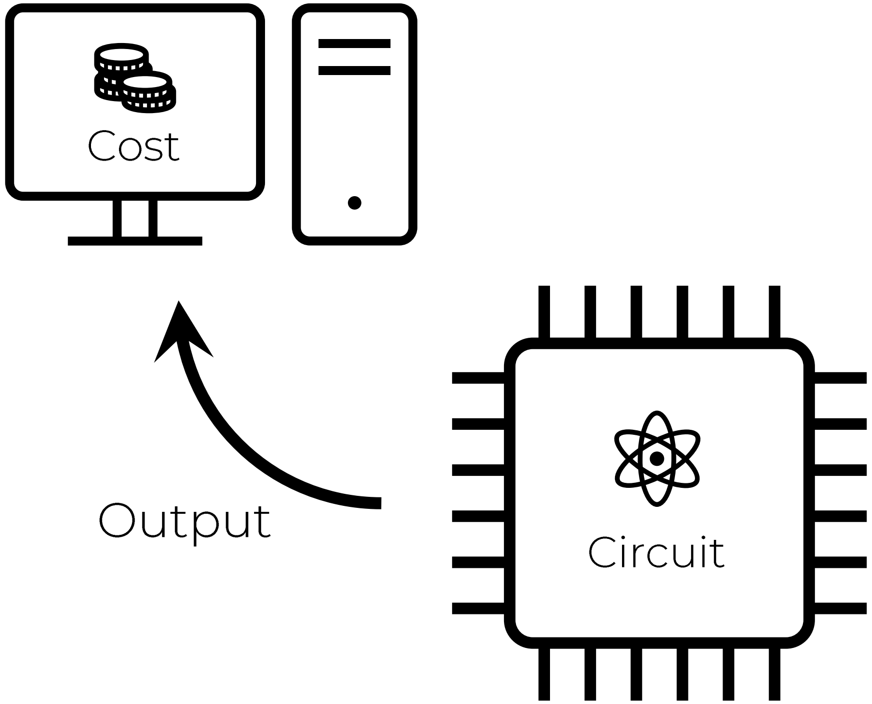

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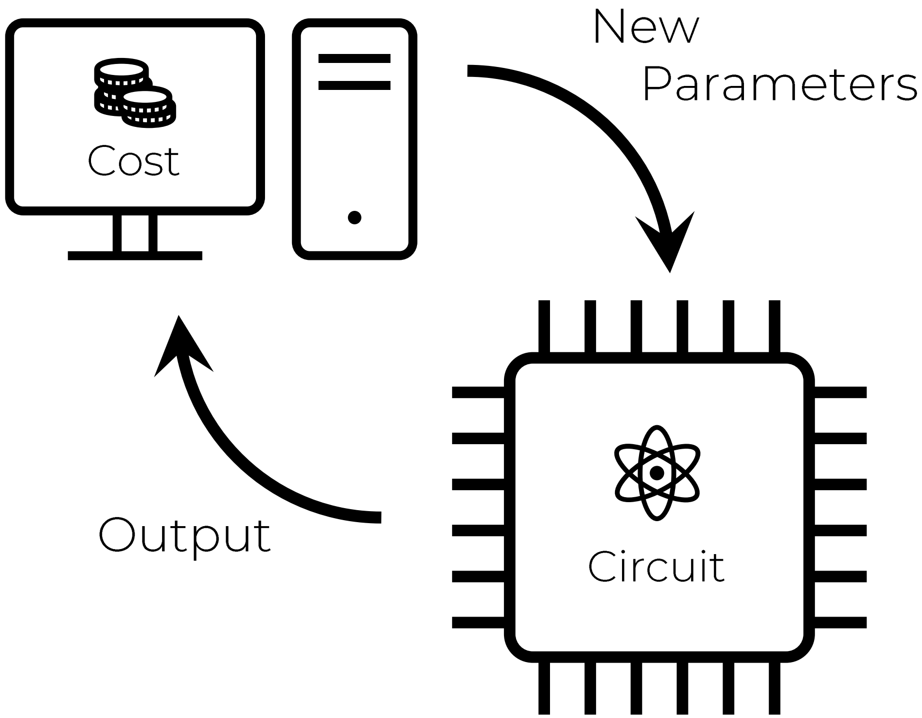

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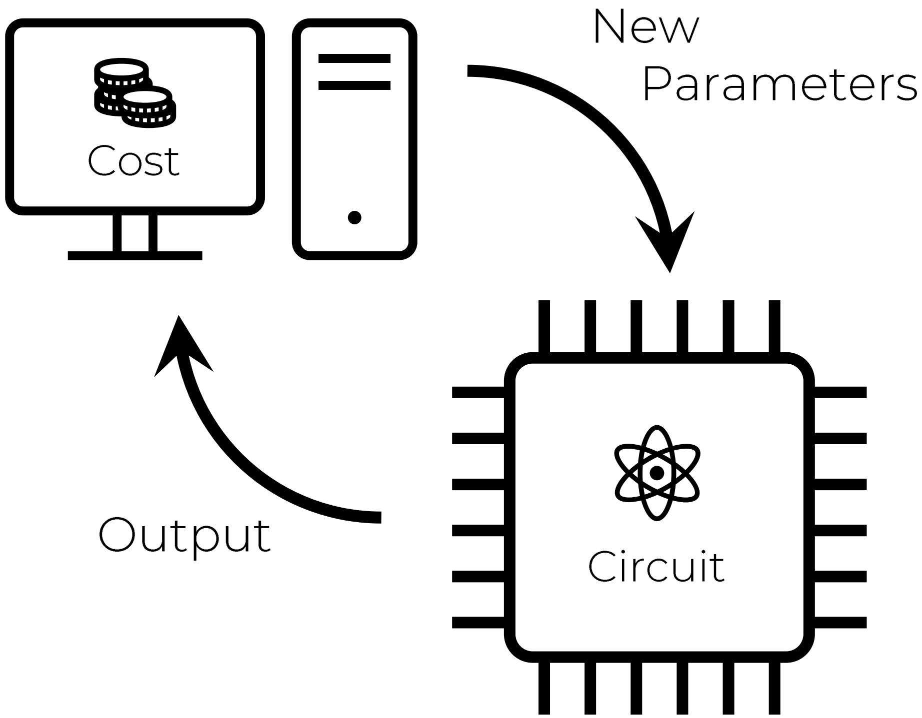

Prior work[1,2] focused on single-parameter metrology and surrogates for the Quantum Fisher Information

[1]Kaubrügger, Raphael, et al. *Physical Review Letters* 123.26 (2019): 260505.
[2]Koczor, Bálint, et al. "Variational-State Quantum Metrology." *New Journal of Physics* (2020).

Cost Function

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Include post-processing by considering a function of the parameters:
Exploit transformation rule of Fisher Information Matrix

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Need a scalar cost function:
Apply weighted trace to both sides of the CRB!

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$$\mathrm{Tr}\{W\,\mathrm{Cov}(\hat{\boldsymbol{f}})\} \geq \frac{1}{n}\,\mathrm{Tr}\{W I_{\boldsymbol{f}}^{-1}\} = \frac{1}{n} C_W$$

Calculation of Fisher Information

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Fisher Information Matrix w.r.t. the physical parameters

$$[I_{\boldsymbol{\phi}}]_{jk} = \sum_l \frac{1}{p_l} \frac{\partial p_l}{\partial \phi_j} \frac{\partial p_l}{\partial \phi_k}$$

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Fisher Information Matrix w.r.t. the physical parameters

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Exploit parameter-shift rule[1,2] to calculate derivatives

$$\partial_j p_l(\boldsymbol{\phi}) = \frac{1}{2} \left[p_l \left(\boldsymbol{\phi} + \frac{\pi}{2} \boldsymbol{e}_j \right) - p_l \left(\boldsymbol{\phi} - \frac{\pi}{2} \boldsymbol{e}_j \right) \right]$$

[1]Schuld, Maria, et al. *Physical Review A* 99.3 (2019): 032331.
[2]Banchi, Leonardo, and Gavin E. Crooks. arXiv preprint arXiv:2005.10299 (2020).

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The classical Fisher information matrix w.r.t. post-processed parameters can be computed using the post-processing‘s Jacobian

The cost function is obtained from a weighted trace of the Cramér-Rao bound

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Multiple extensions of the algorithm, for example including prior knowlege (Bayesian approach)

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Details on the implementation of parameter-shift rules

Numerical experiments that showcase the performance of the approach

Take-Home Message

> Near-term quantum computers can be used to design the next generation of quantum sensors

Thank you for your attention!

@jj_xyz

Paper

Demo

Slides

The Algorithm Landscape

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Single Parameter

Multiparameter

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Kaubrügger et al.

COST FUNCTION
Spin Squeezing

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Fixed Circuit

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The Algorithm Landscape

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COST FUNCTION
Fidelity

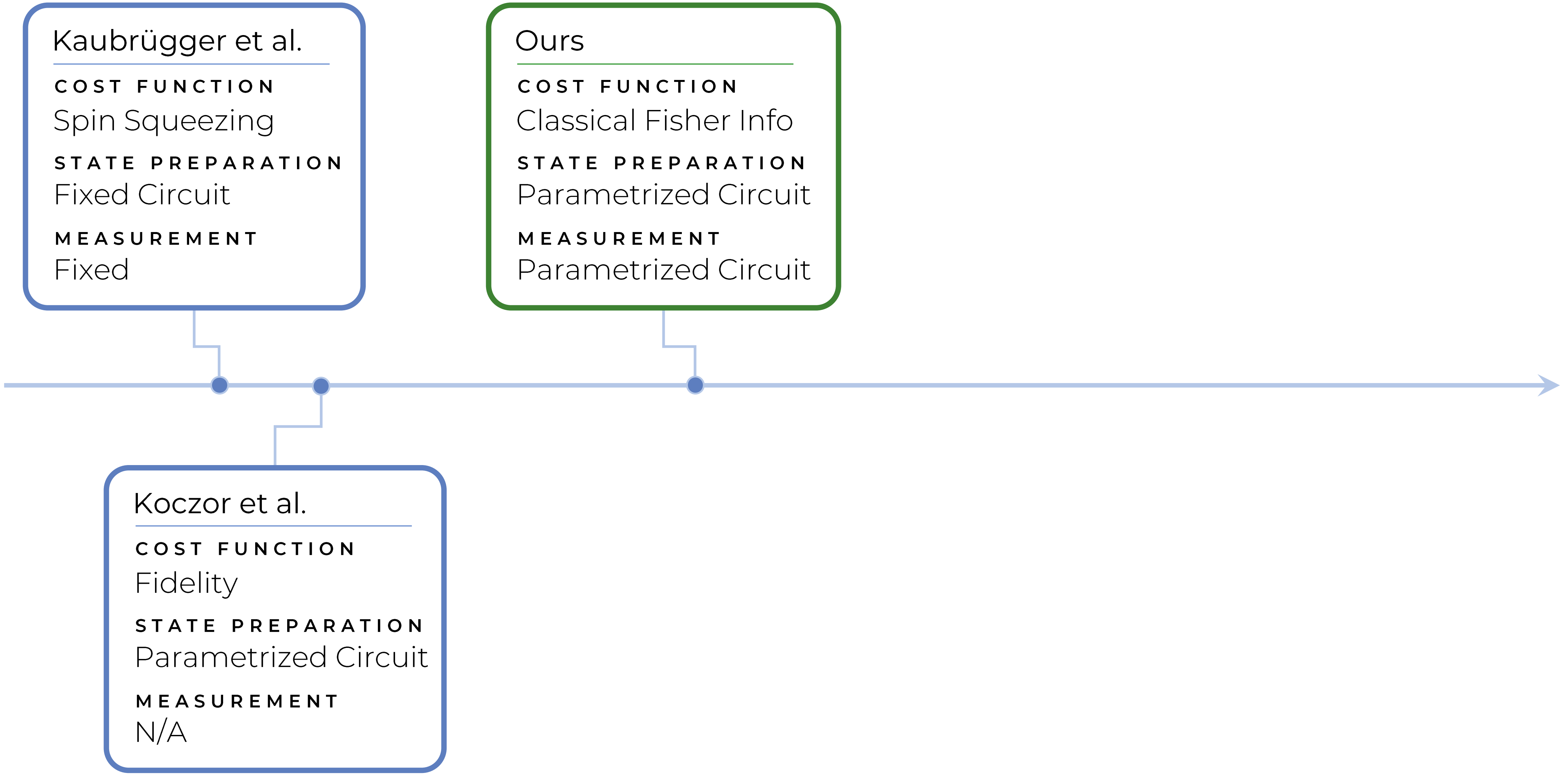
STATE PREPARATION
Parametrized Circuit

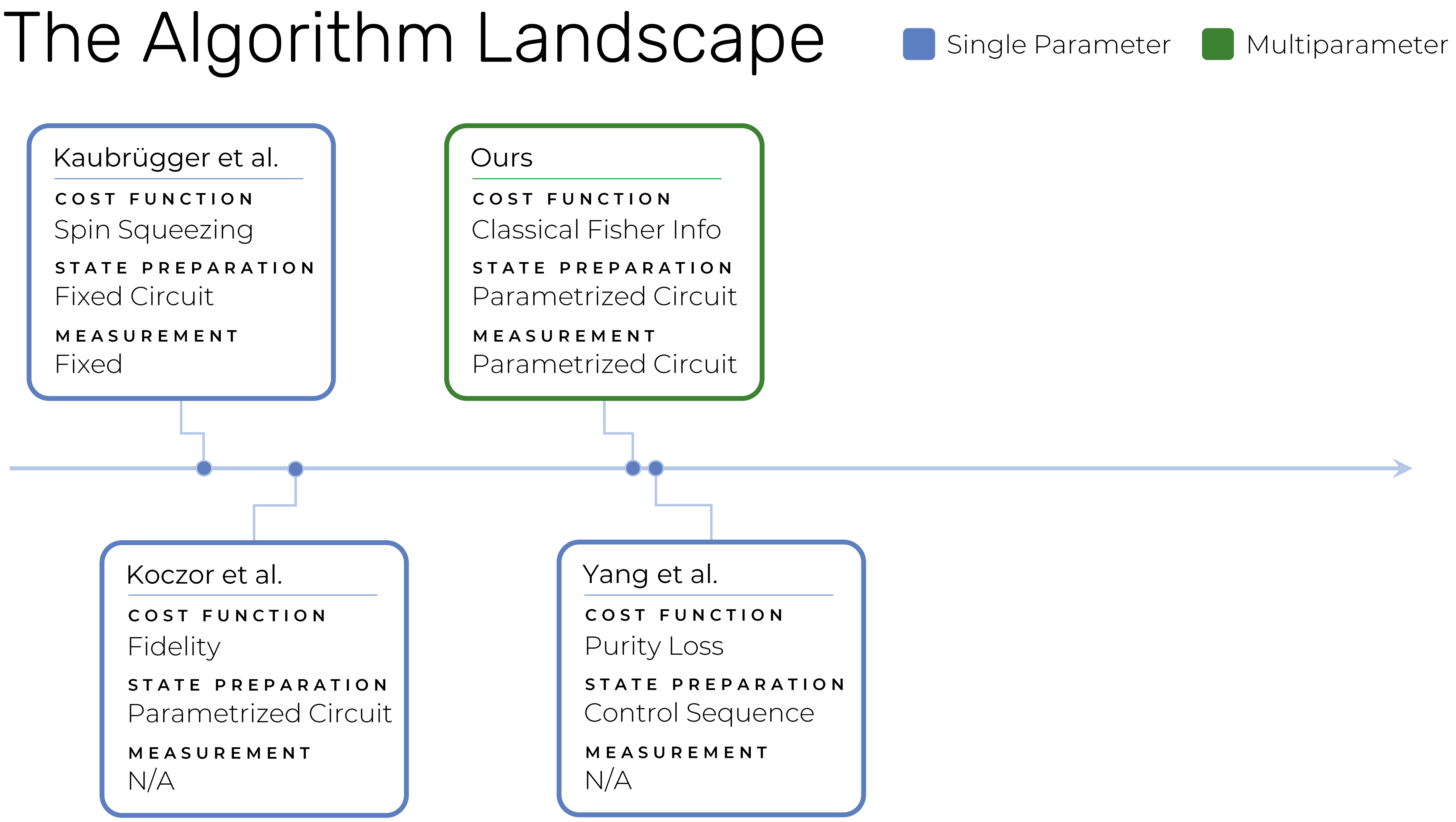
MEASUREMENT
N/A

The Algorithm Landscape

Single Parameter

Multiparameter

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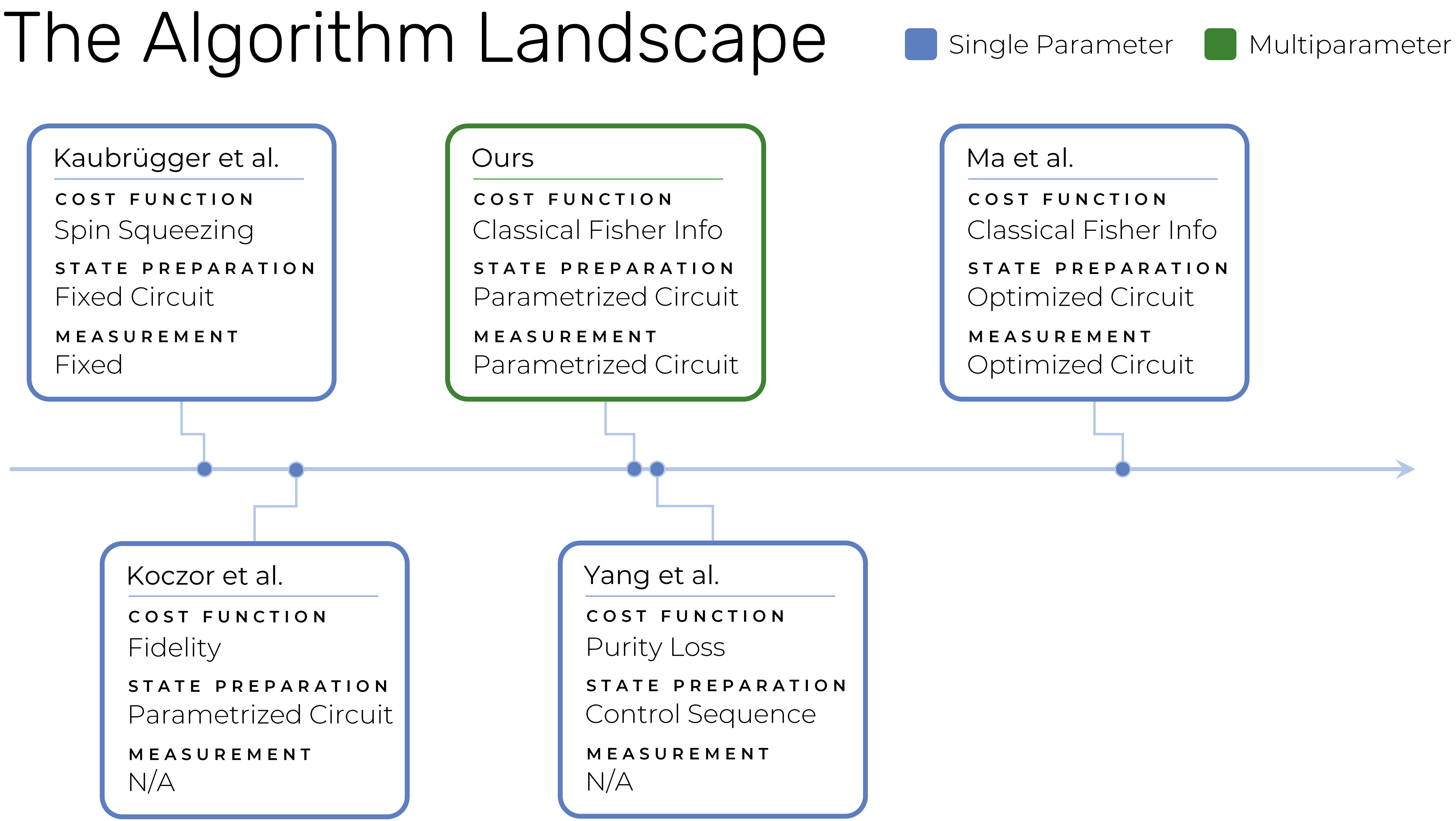
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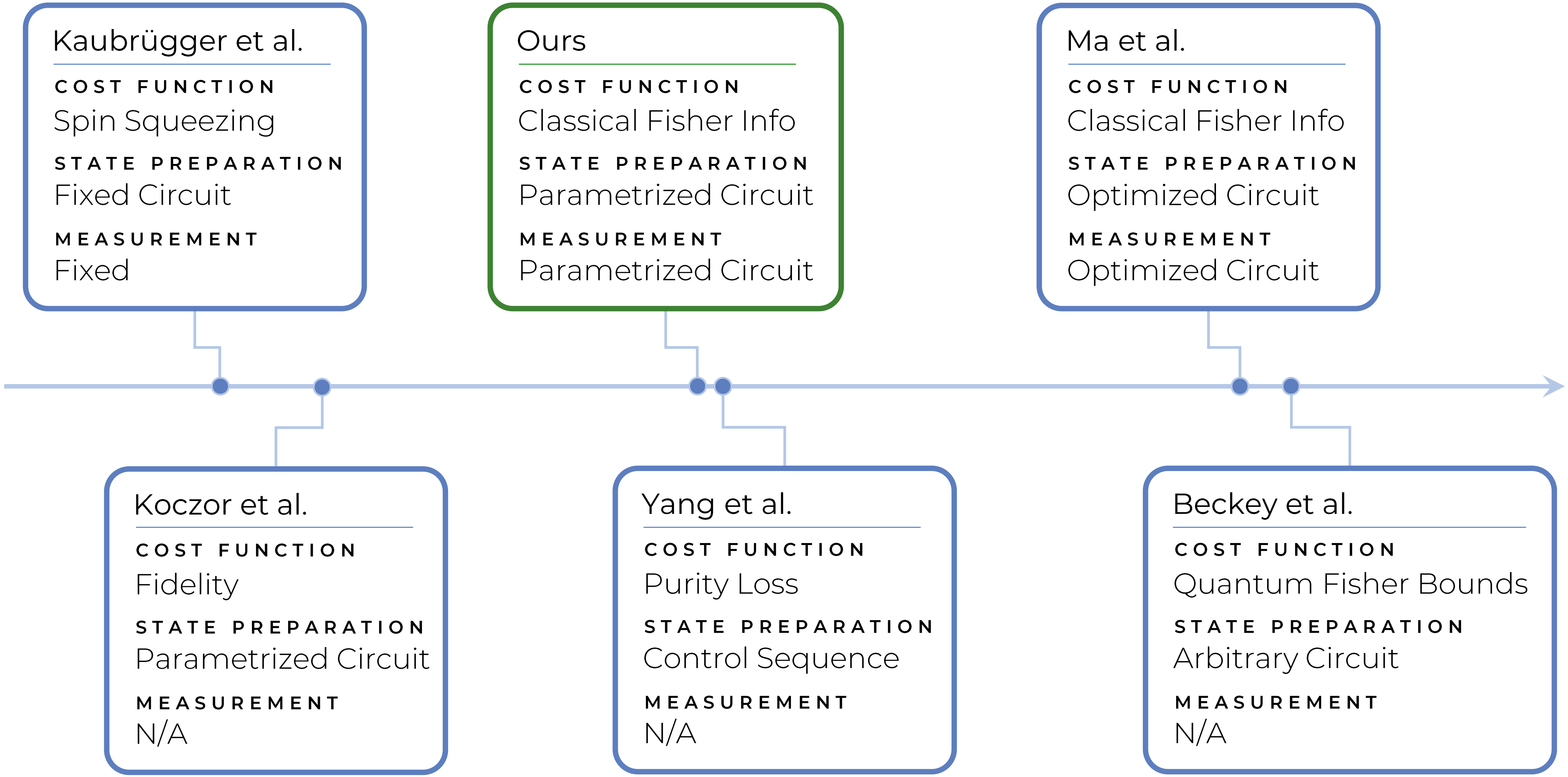
Single Parameter   Multiparameter



Parameter-Shift Rule

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