IOP QUANTUM 2020

Improving Quantum Sensing with Quantum Computers

JOHANNES JAKOB MEYER, FU BERLIN

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A variational toolbox for quantum multi-parameter estimation

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(Dated: June 11, 2020)

Johannes Borregaard
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Jens Eisert
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Quantum Metrology

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Physical quantities (magnetic fields, energies, ...)
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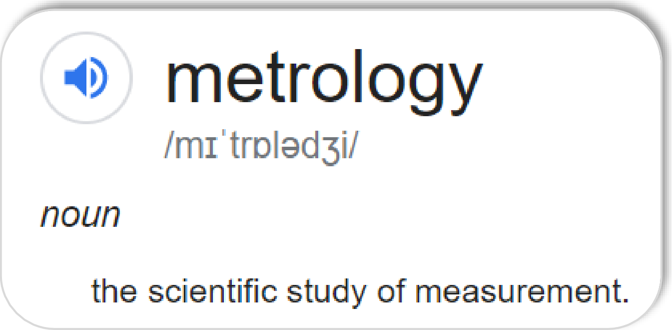

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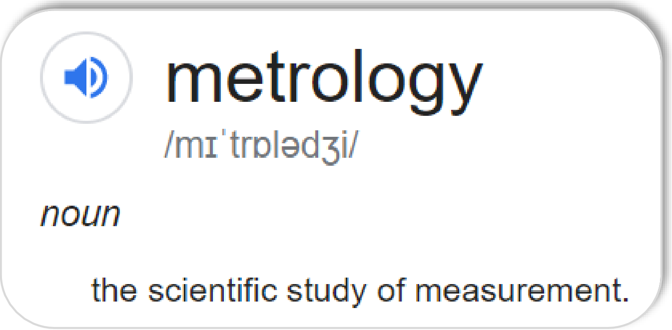

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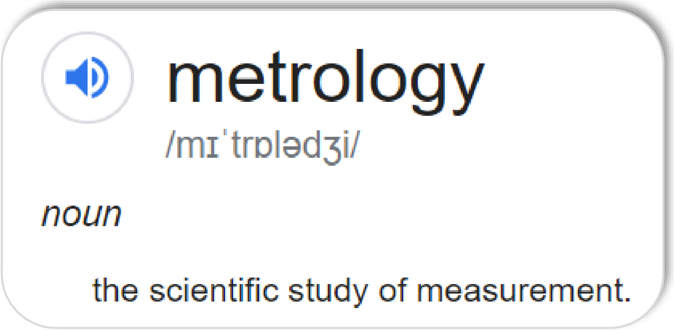

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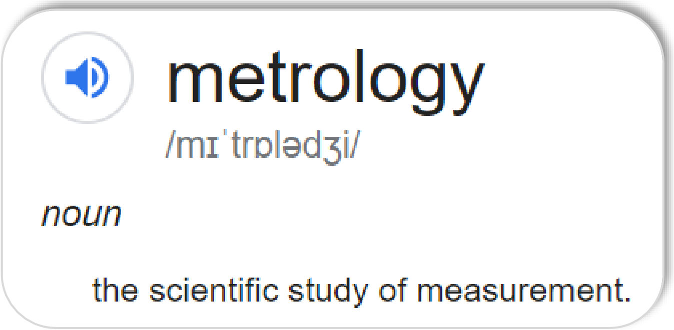

Probe
State

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noun
the scientific study of measurement.

Probe State		Encoding Evolution
ρ	$\longrightarrow$	$\mathcal{E}(\phi)$

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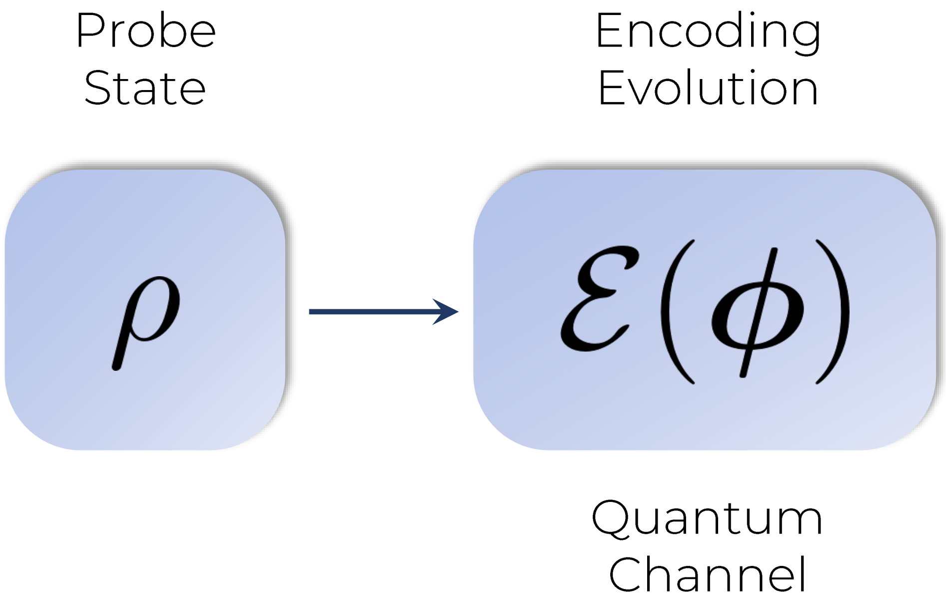

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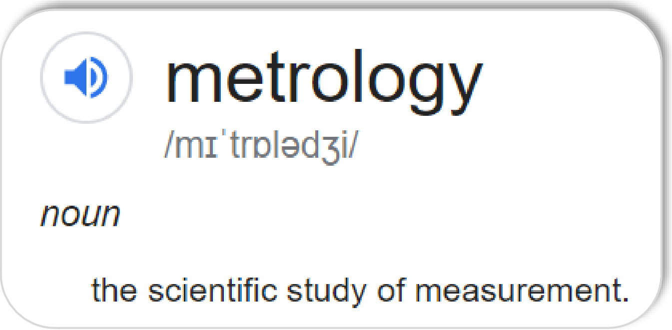

Probe State: ρ

$\rightarrow$

Encoding Evolution: $\mathcal{E}(\phi)$ (Quantum Channel)

$\rightarrow$

Measurement: $\{\Pi_l\}$

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Probe State	Encoding Evolution	Measurement	Output Distribution
ρ	$\mathcal{E}(\phi)$	$\{\Pi_l\}$	$p_l(\phi)$
	Quantum Channel	POVM	

$$\rho \longrightarrow \mathcal{E}(\phi) \longrightarrow \{\Pi_l\} \dashrightarrow p_l(\phi)$$

Cramér-Rao Bound

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Task: Compute an **estimator** from the output distribution

$$p_l(\boldsymbol{\phi}) \;\rightarrow\; \hat{\boldsymbol{\varphi}} \colon \mathbb{E}\{\hat{\boldsymbol{\varphi}}\} = \boldsymbol{\phi}$$

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$$\mathrm{Cov}(\hat{\boldsymbol{\varphi}}) \geq \frac{1}{n} I_{\boldsymbol{\phi}}^{-1}(\mathrm{POVM})$$

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Pr $\boxed{\mathrm{Tr}\{\mathrm{Cov}(\hat{\varphi})\} = \mathrm{MSE}(\hat{\varphi})}$ **Cramér-Rao bound**

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We need to find optimal probes and measurements

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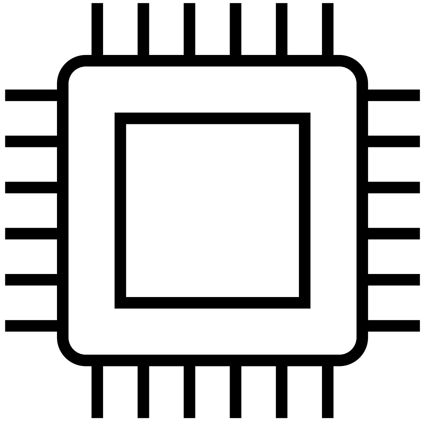

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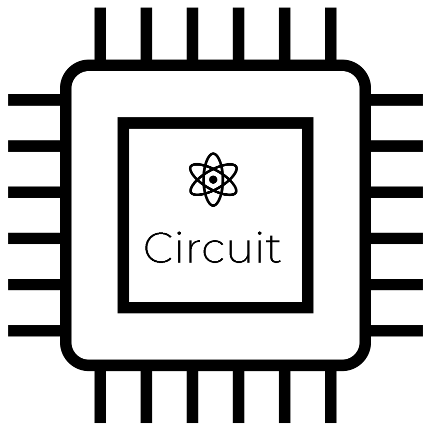

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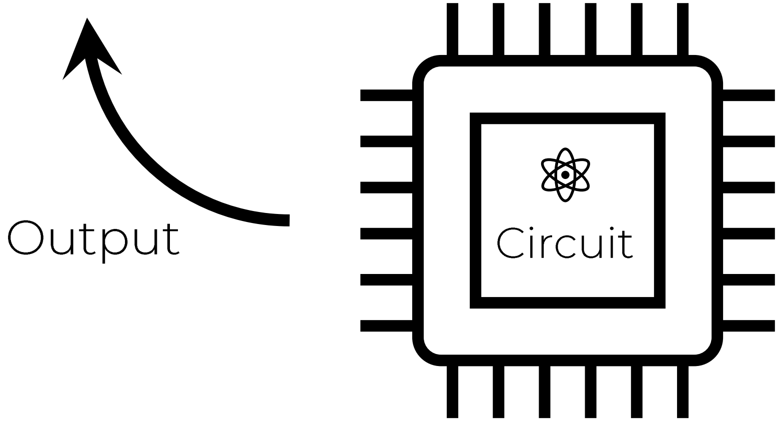

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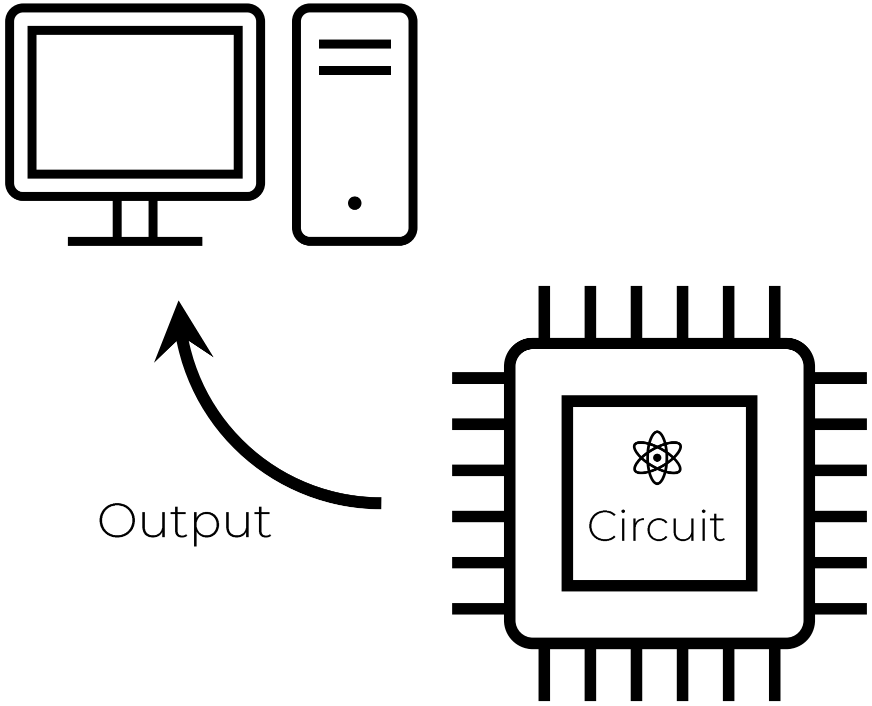

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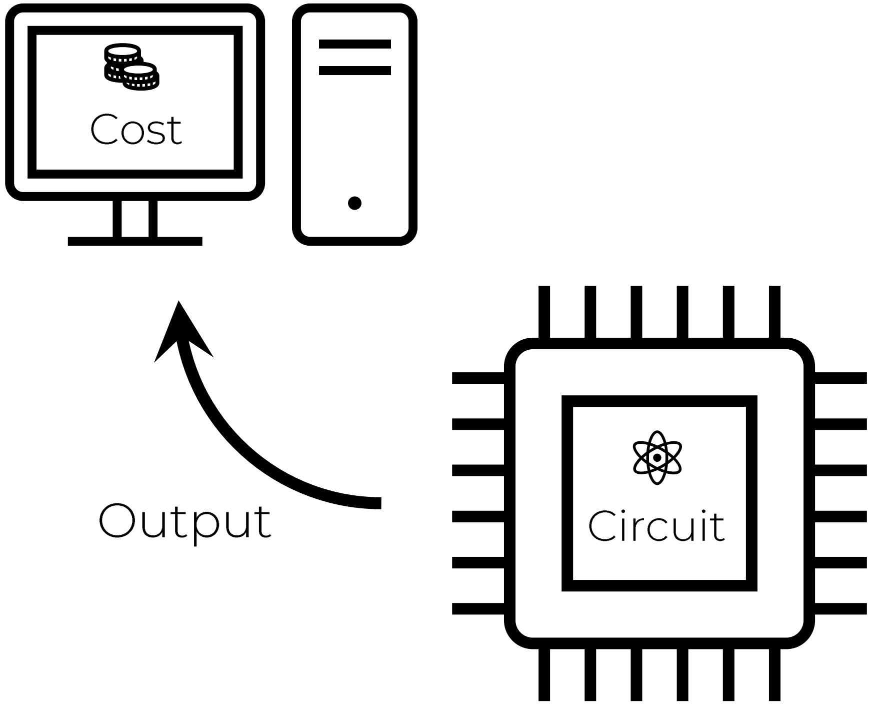

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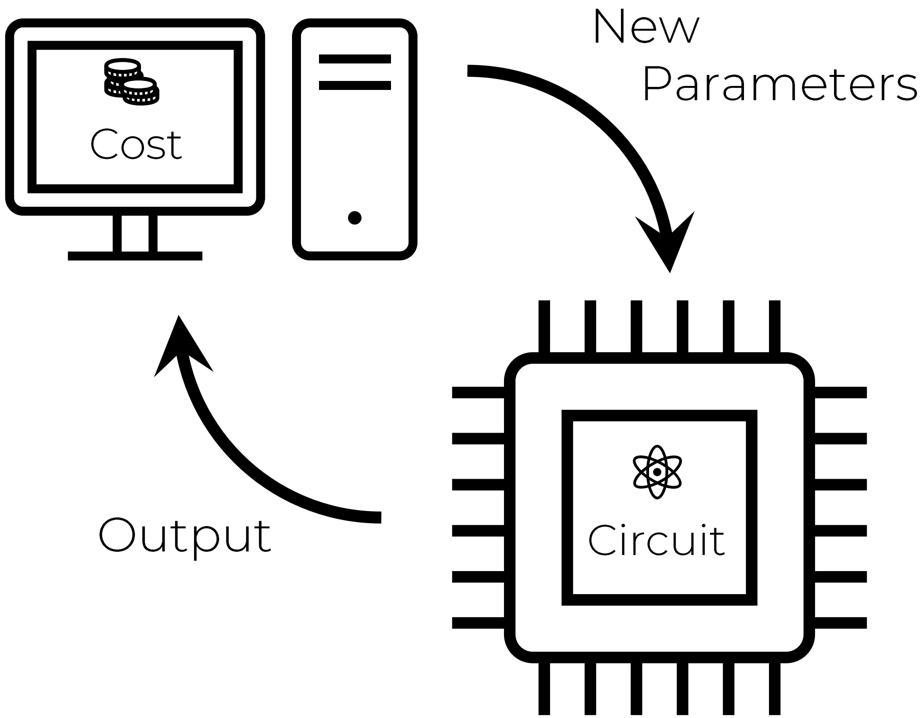

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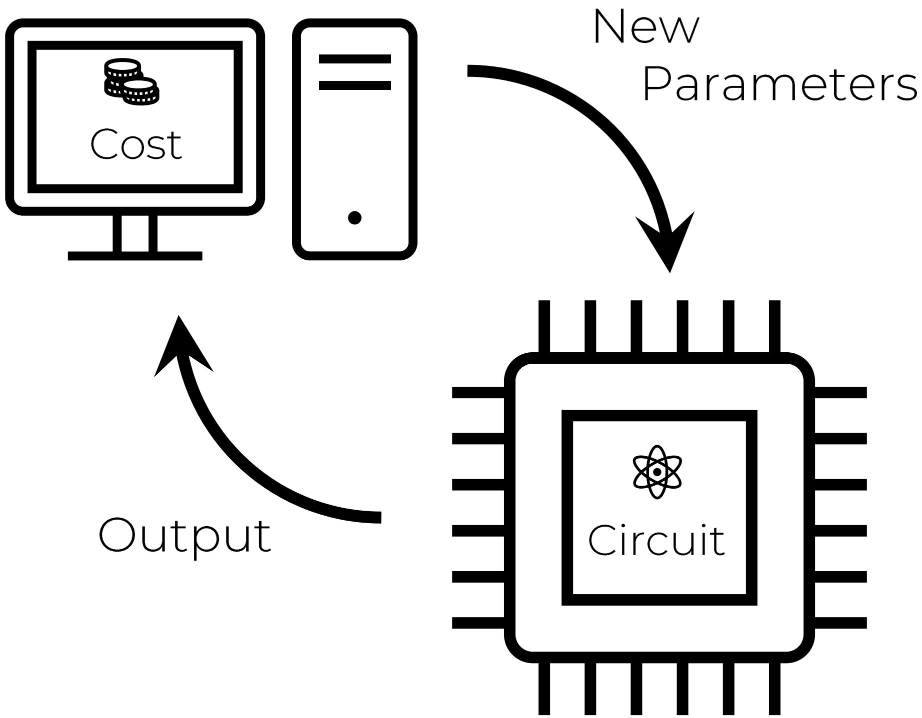
Prior work[1,2,3] focused on single-parameter metrology and surrogates for the Quantum Fisher Information



[1]Kaubruegger, Raphael, et al. *Physical Review Letters* 123.26 (2019): 260505.
[2]Koczor, Bálint, et al. "Variational-State Quantum Metrology." *New Journal of Physics* (2020).
[3]Yang, Xiaodong, et al. *npj Quantum Information* 6.1 (2020): 1-7.

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We consider the sensing protocol as an entire prescription

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→ Include probe state, measurement and post-processing

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Quantify the quality with the classical Cramér-Rao bound

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Generalize to the multi-parameter setting

The Algorithm

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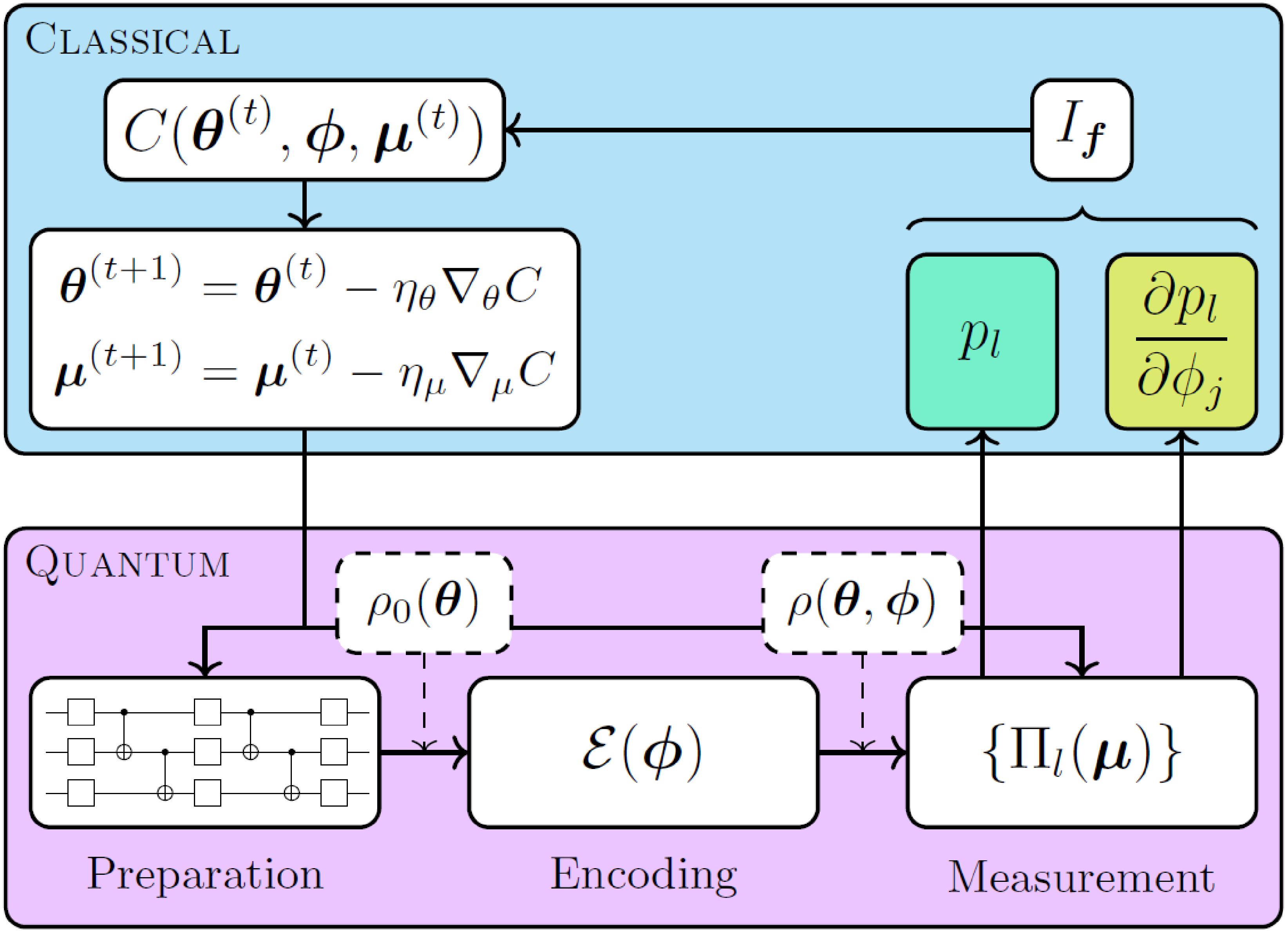

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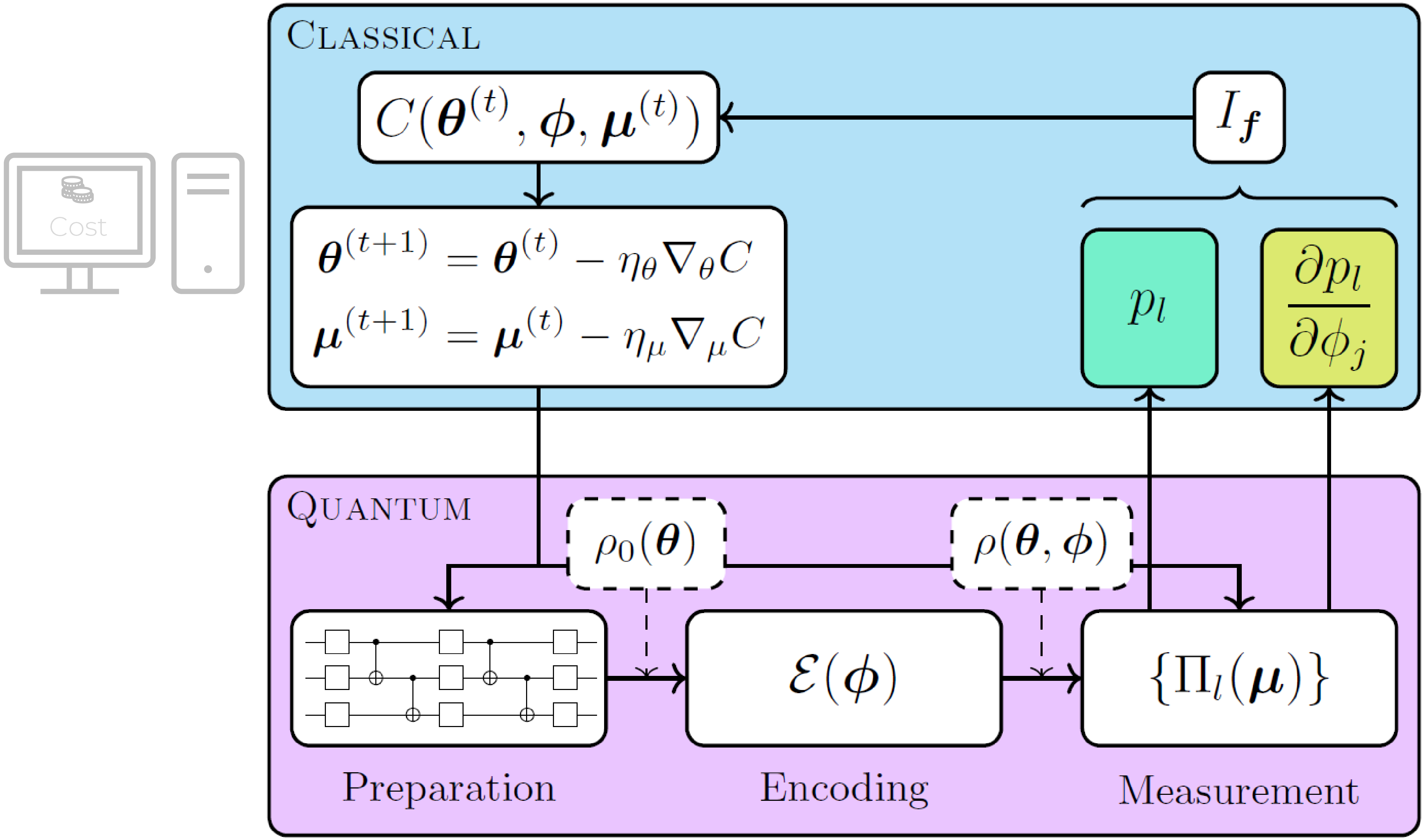

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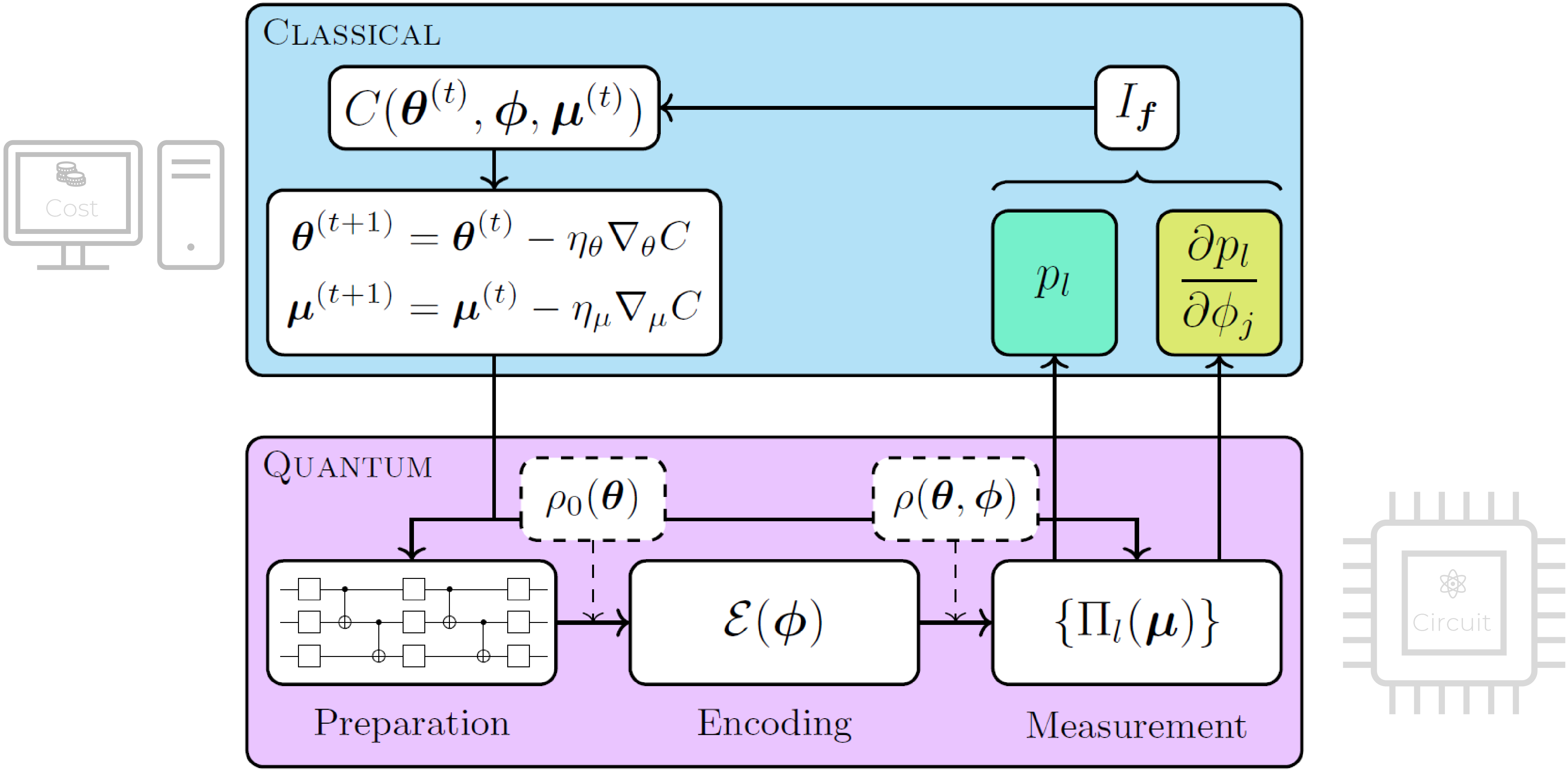

Calculation of Fisher Information

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Fisher Information Matrix w.r.t. the physical parameters

$$[I_{\boldsymbol{\phi}}]_{jk} = \sum_l \frac{(\partial_j p_l)(\partial_k p_l)}{p_l} \quad \partial_j = \frac{\partial}{\partial \phi_j}$$

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Exploit parameter-shift rule[1,2] to calculate derivatives

$$\partial_j p_l(\boldsymbol{\phi}) = \frac{1}{2}\left[p_l\left(\boldsymbol{\phi} + \frac{\pi}{2}\boldsymbol{e}_j\right) - p_l\left(\boldsymbol{\phi} - \frac{\pi}{2}\boldsymbol{e}_j\right)\right]$$

[1]Schuld, Maria, et al. *Physical Review A* 99.3 (2019): 032331.
[2]Banchi, Leonardo, and Gavin E. Crooks. arXiv preprint arXiv:2005.10299 (2020).

Cost Function

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Exploit transformation rule of Fisher Information Matrix

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Need a scalar cost function:
Apply weighted trace to both sides of the CRB!

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$$\boxed{\begin{aligned}\mathrm{Tr}\{W\,\mathrm{Cov}(\hat{\boldsymbol{\varphi}})\} &= \mathrm{MSE}_W(\hat{\boldsymbol{\varphi}}) \\ &= \mathbb{E}\{\langle \hat{\boldsymbol{\varphi}} - \boldsymbol{\phi}, W(\hat{\boldsymbol{\varphi}} - \boldsymbol{\phi})\rangle\}\end{aligned}}$$

N
A

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- Numerical Experiments
- Extensions of the algorithm
- A parameter-shift rule for noise channels

Take-Home Message

> Near-term quantum computers can be used to design the next generation of quantum sensors

Thank you for your attention!

Paper

Demo

Slides