IOP QUANTUM 2020

What Functions can Quantum Models Learn?

JOHANNES JAKOB MEYER, FU BERLIN

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The effect of data encoding on the expressive power of variational quantum machine learning models

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Disclaimer: After uploading to the arxiv we were notified of work by Vidal and Theis[1] that has significant overlap with ours.

[1]Gil Vidal, Francisco Javier, and Dirk Oliver Theis. "Input Redundancy for Parameterized Quantum Circuits." *Frontiers in Physics* 8 (2020): 297.

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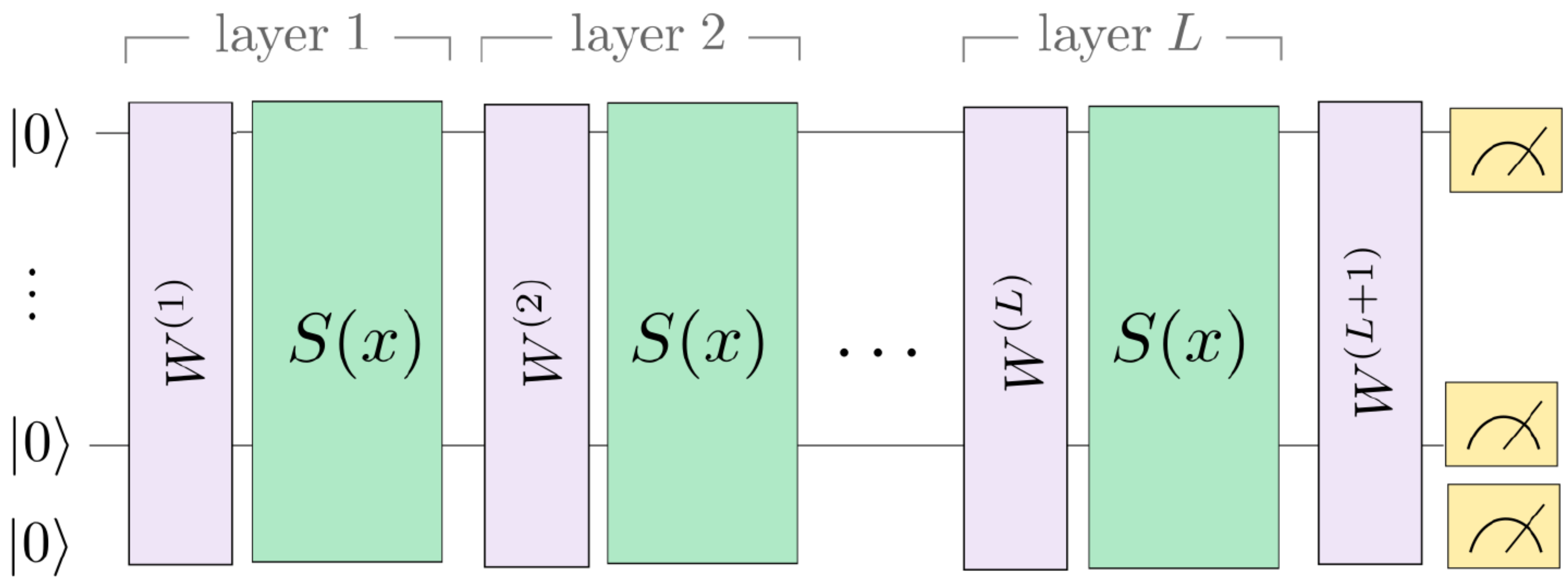
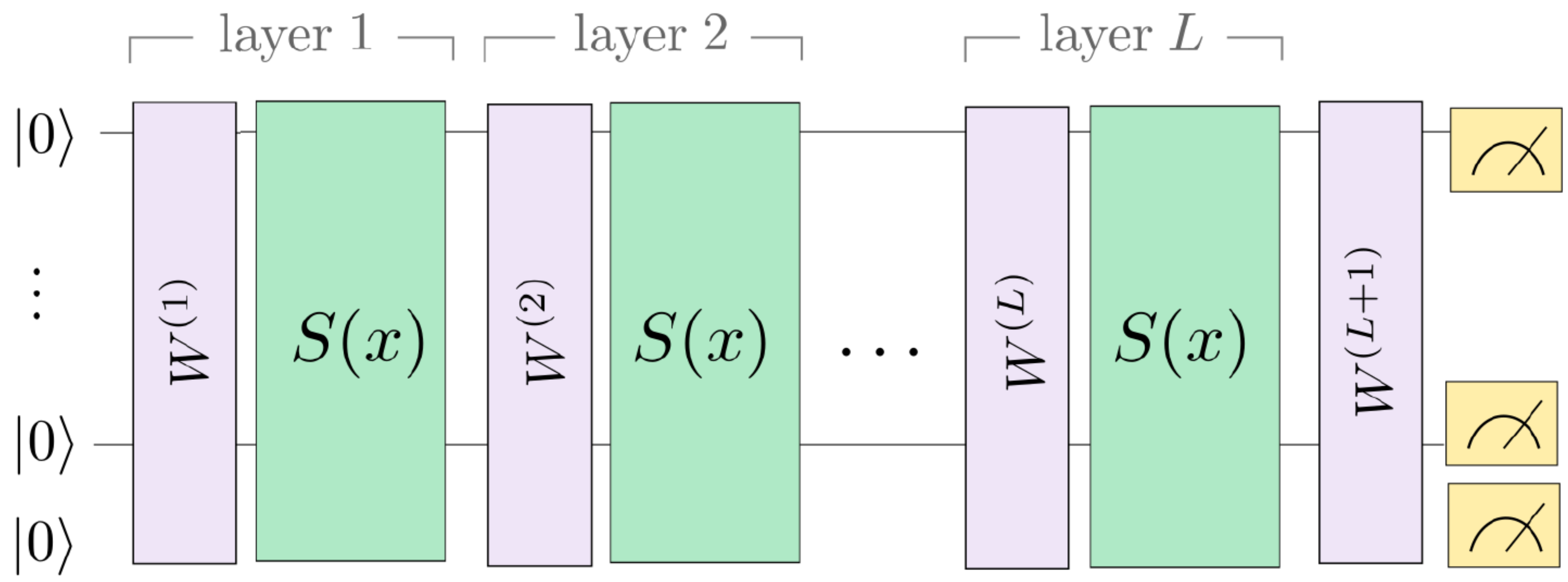
A lot of work has been done to understand the practical side, but very little is known on the theory side

So we asked ourselves: **What functions can such models learn?**

Variational Quantum Learning Models



Variational Quantum Learning Models



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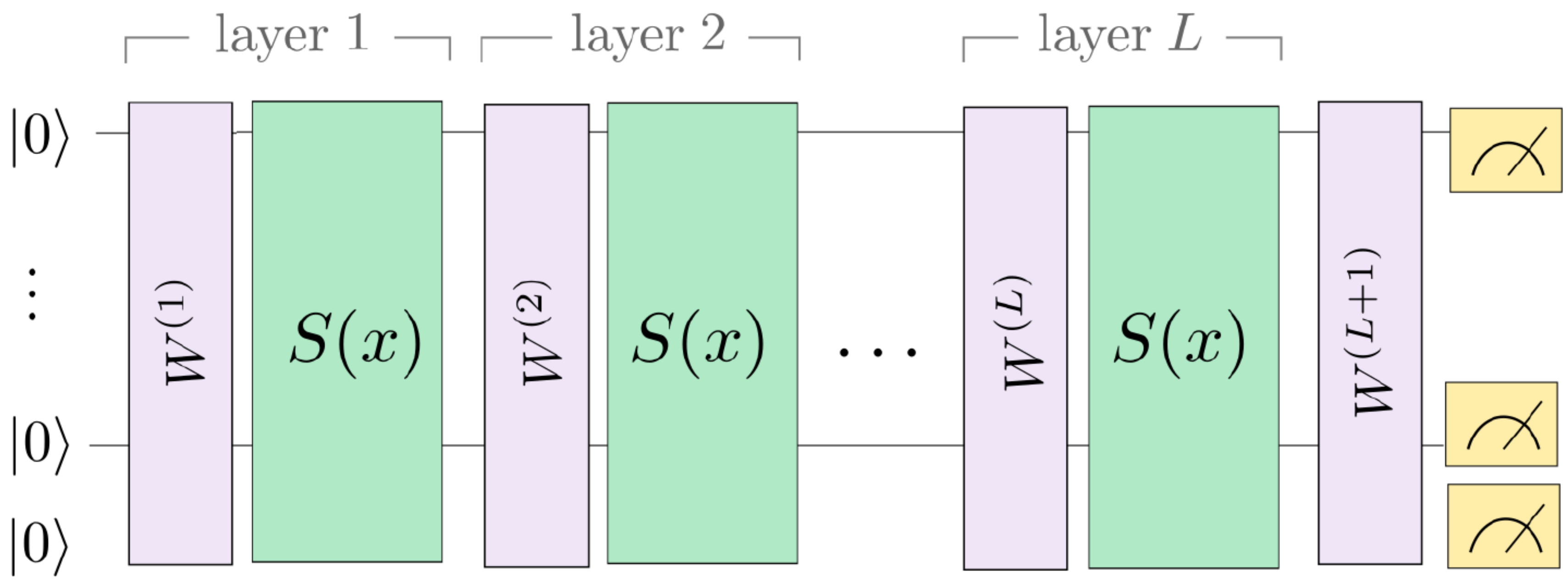
Model Output

Variational Quantum Learning Models



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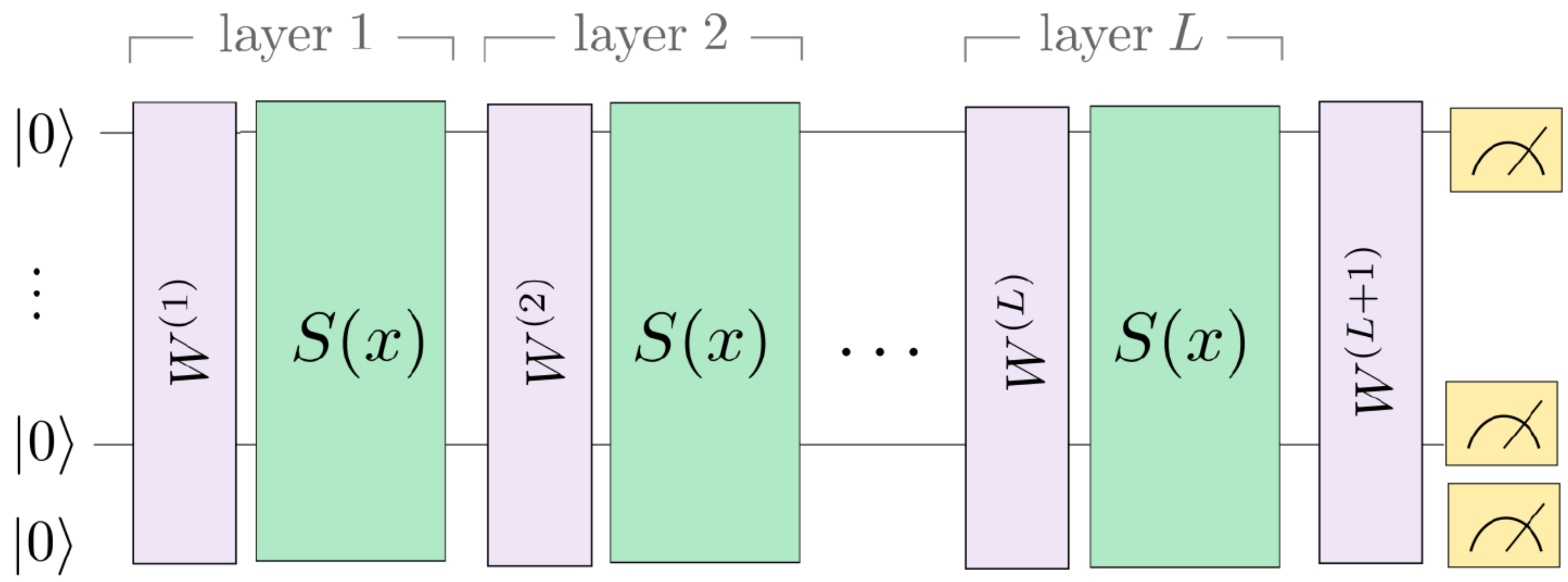
$$S(x) = e^{-ixH}$$

Hamiltonian Evolution

Variational Quantum Learning Models



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Hamiltonian Evolution

$$W^{(l)}(\boldsymbol{\theta})$$

Trainable Blocks

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The output state contains all possible sums of frequencies

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Output is expectation value and therefore contains a complex conjugation

$$f_{M,\boldsymbol{\theta}}(x) = \langle \psi_{\boldsymbol{\theta}}(x) | M | \psi_{\boldsymbol{\theta}}(x) \rangle = \sum_{\omega \in \Omega} c_\omega(M, \boldsymbol{\theta}) e^{i\omega x}$$

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$$\Omega = \{\lambda_j - \lambda_k \,|\, \lambda_j, \lambda_k \in \mathrm{spec}(H)\}$$

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$$\Omega = \{\lambda_{j_1} + \cdots + \lambda_{j_L} - \lambda_{k_1} - \cdots - \lambda_{k_L} \,|\, \lambda_{j_l}, \lambda_{k_l} \in \mathrm{spec}(H)\}$$

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The accessible spectrum consists of all sums of differences of eigenvalues of the generator of the data encoding

Take-Home Message #1

> Quantum learning models output Fourier series, repeating data encoding gives access to higher frequencies

Pauli-Encodings

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The number of available frequencies grows linearly in depth and width

For general encodings the dependence can be exponential:

$$\#(\text{frequencies}) \leq \frac{d^{2L}}{2} - 1$$

d System dimension

L Number of layers

Consequences for Learning Tasks

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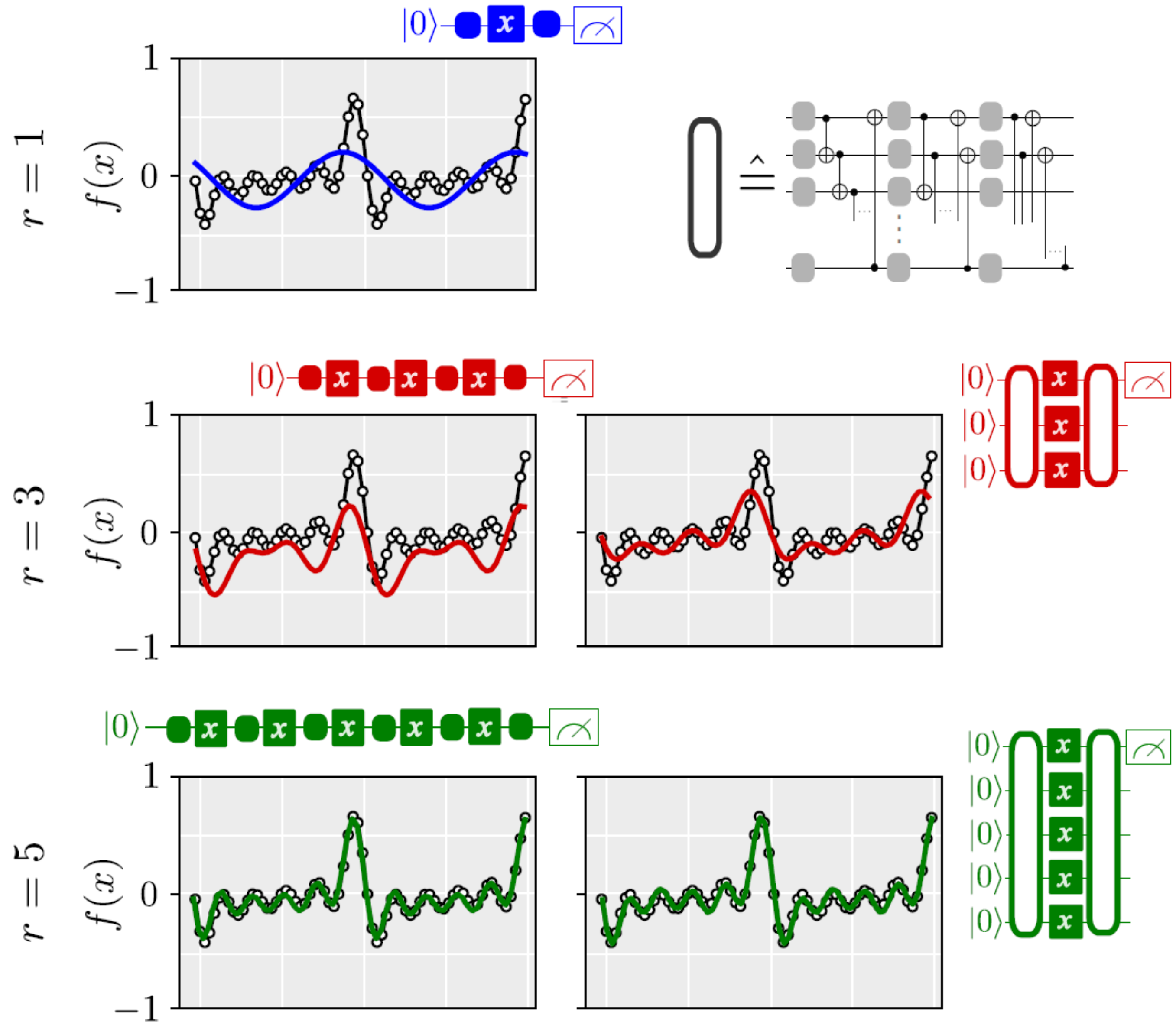

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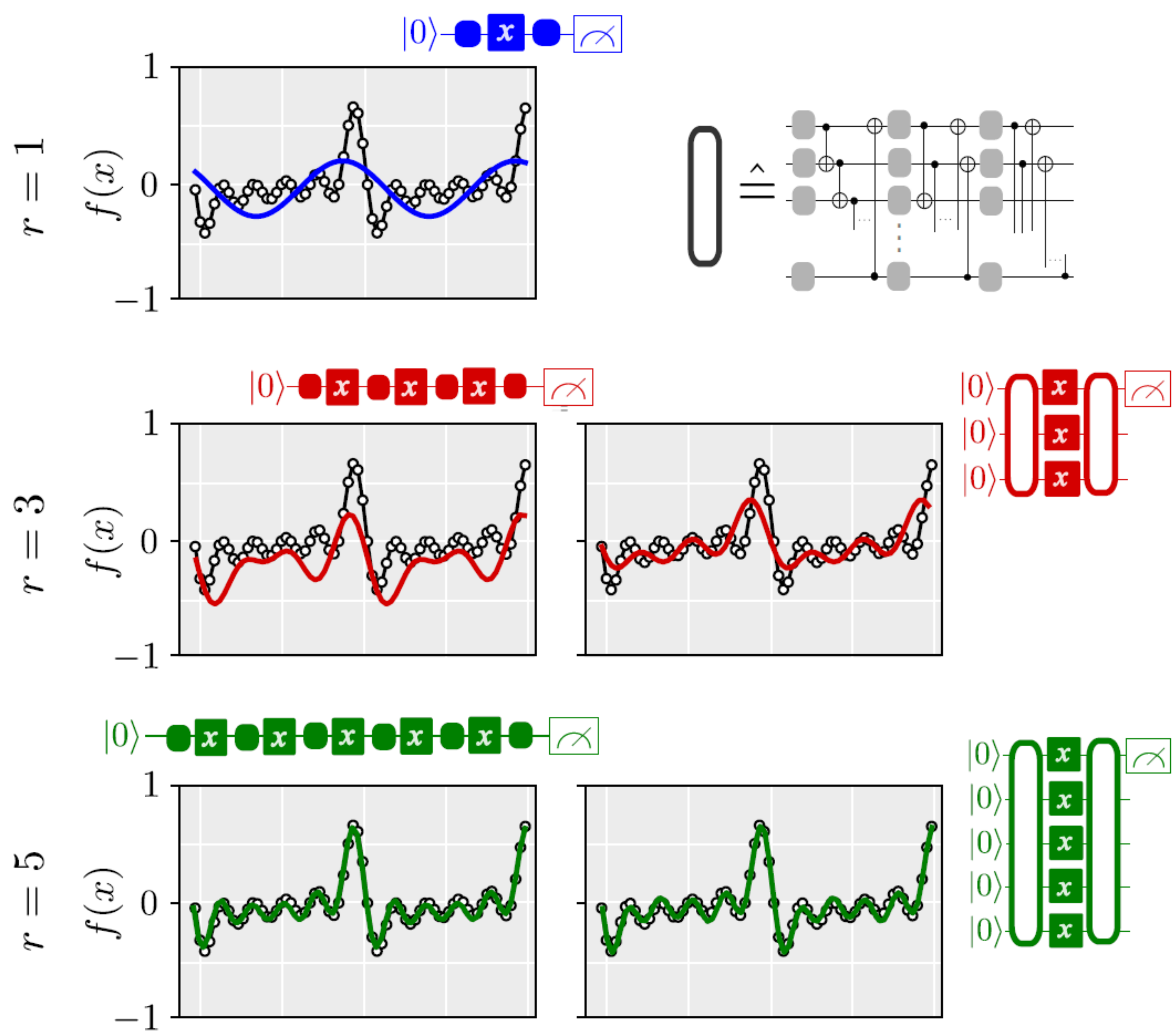

You can reproduce all figures from the paper at home!

Universality of Quantum Models

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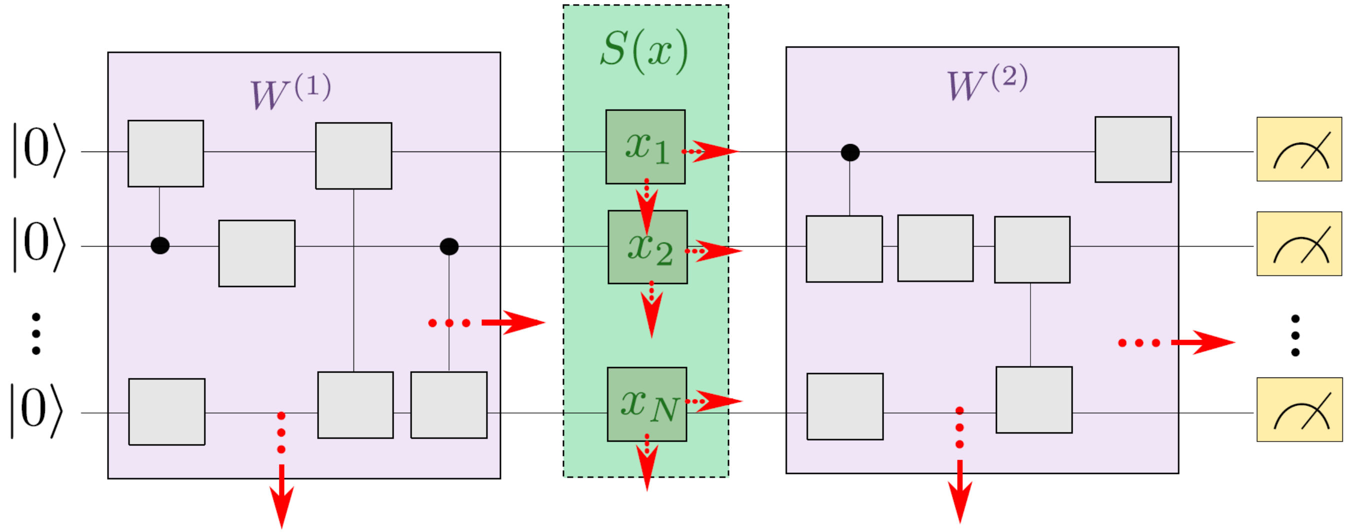
A model with one layer of data encodings generated by a *universal Hamiltonian family* and arbitrary unitaries is a universal function approximator



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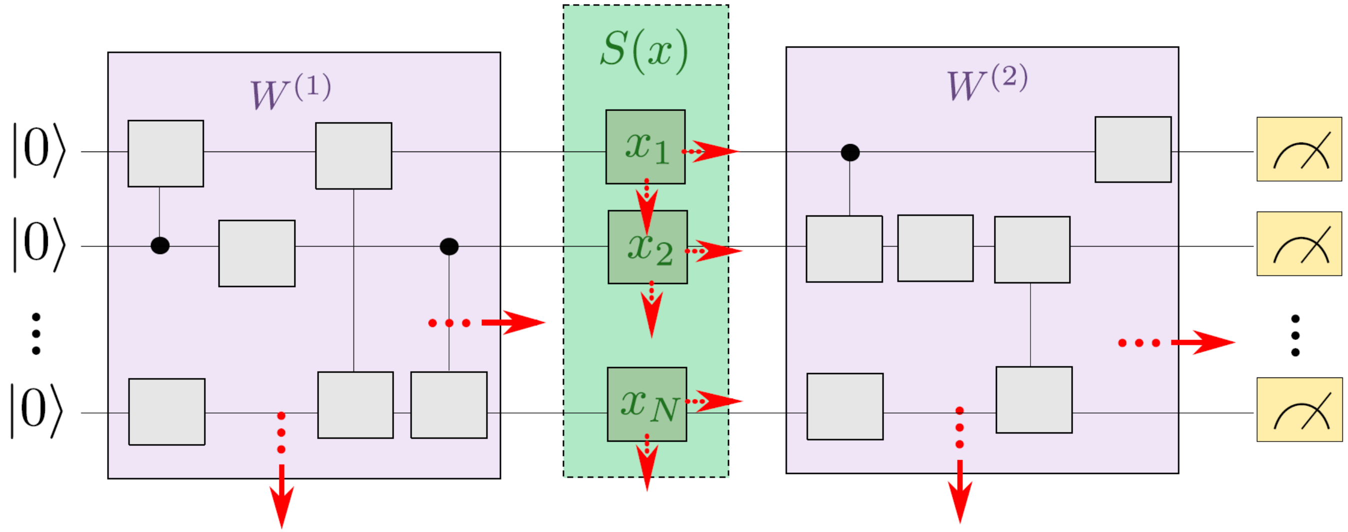

A universal Hamiltonian family asymptotically has access to all integer frequencies. Repeated single-qubit Pauli rotation encodings are a universal Hamiltonian family!

Take-Home Message #2

> Quantum learning models are universal function approximators

Practical Implications for Model Building

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5. Make your observables trainable

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3. Is universal approximation possible with fixed qubit numbers?
4. Are quantum models good for signal processing?

Thank you for your attention!

Paper

Demo

Slides